Vendor-Buyer Supply Chain Management models with and without backorder - an inspection at vendor site

Doctoral Thesis submitted
In partial fulfillment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY

In

MANAGEMENT

By

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December 2019

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ACKNOWLEDGEMENT

The journey of pursuing a Ph.D. with ICFAI University Jharkhand has been challenging, exciting and a novel experience. The continuous progress of the work was backed with the support and encouragement of numerous people. This humble work has finally seen through its completion due to the immense motivation received from everyone. I would like to thank all those who helped in making this possible and an unforgettable experience for me.

The culmination of my work extends with a pleasant task of expressing my thanks to all those who contributed to the success of this study. At this moment of accomplishment, first I express my deep sense of gratitude to **Dr. Bal Mukund Singh**, Ex-Dean and Registrar, ICFAI University, Jharkhand, and my research supervisor **Dr. Shree Ranjan**, Head of Department IT, ICFAI University, Jharkhand, for his able supervision in my research work. He was very helpful in discussing the matters pertaining to the study and offered constructive suggestions during the course of the work. He always guided and motivated me. Without his guidance, writing thesis would have been possible. I remain thankful for all his effort in making it a reality.

I am extremely indebted to my co-supervisor **Dr. Danish Ali Khan**, Professor, Department of Computer Application, National Institute of Technology, Jamshedpur, for his untiring support and guidance. I could not have ever dreamt about supply chain research without his strong support, guidance and co-operation. I have learnt a lot regarding supply chain research and thesis writing under his watchful guidance. Words are inadequate to express my deep sense of gratitude to him for his valuable supervision.

I do not have words to thank **Prof. O. R. S. Rao**, Vice-Chancellor, ICFAI University Jharkhand for his continuous motivation to me to complete my research work and guiding the path to complete my research work.

My research work could not be completed without help of **Dr. Tarni Mondal**, Professor and Head of Department, Mathematics Department, National Institute of Technology, Jamshedpur. He guided me to learn the theory of expectation, renewal and reward theorem, etc. He cross-checked all mathematical derivations of my research work. I express my deep sense of gratitude to him as well.

I offer my sincere gratitude to all the faculties of the ICFAI University, Jharkhand. I would also like to convey my sincere thanks to Dr. Hariharan and Dr. Rumna Bhattacharya for their continuous support and guidance.

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ABSTRACT

Every business has to maintain some sort of inventory and henceforth inventory management becomes one of the important activities. The major activities of inventory management are the flow of goods and the flow of information. The goods move down-stream from the supplier to the manufacturer, the manufacturer to the buyer (passing through intermediaries like wholesaler/stockiest/dealer) and the buyer to the consumer. Information, initially generated from the customer in the form of demand, flows up-stream to the buyer (dealer), from the buyer to the manufacturer and finally from the manufacturer to suppliers of raw materials.

Industries that are involved in manufacturing of items need to cut the cost of their operations because of tough competition in the market. The supply chain management is not just storing goods in an inventory, but it also deals with scheduling production of items, location of inventories, when to place an order, what is the quantity of items to order, how much quantity of items to be shipped, frequency of shipments and many more activities with an objective to minimize the total expected cost of these operations. Research work done in supply chain management was aimed for reducing the total expected cost of inventory management and guided industries to plan their activities accordingly. During 1960-70, the concepts and principles like JIT (Just in Time), was developed in Japan and particularly used by Toyota. This concept played an important role in effective supply chain management.

(Goyal, 1977) had initiated research work on integrated inventory management by giving “An integrated inventory model for a single supplier single customer problem”. The research work was followed by (Banerjee, 1986) “A joint economic-lot-size model for purchaser and vendor” who had given Joint Economic-Lot-Size (JELS). (Lu, 1995) discussed one vendor multi-buyer inventory management. (Salameh & Jaber, 2000) had discussed inventory management with imperfect production quality. (Cárdenas-Barrón, 2000) had corrected (Salameh & Jaber, 2000) formula for calculation of economic order quantity (EOQ) and (Wee et al., 2007) had further extended (Salameh & Jaber, 2000) work by considering permissible shortage backordering. (Khan et al., 2011) extended (Salameh & Jaber, 2000) research work by considering the inspection process errors. (Hsu & Hsu, 2012b) had extended (Wee et al., 2007) and given inventory management for imperfect production quality and imperfect inspection with shortage backordering.

In the earlier research work, including the above mentioned research works, it was mentioned in manufacturing industry the buyer conduct 100% inspection of items after arrival of fresh lot of items. This has been identified as a gap in research works. This research work focus on development of vendor-buyer supply chain management model with or without backorder where inspection is being conducted by the vendor along with production of items. The production process had been considered to be imperfect and could produce some defective items during the production.

Following are objectives the research work in this thesis.

- Identify impact on total cost of the supply chain management when inspection is being performed at the vendor site for imperfect production quality and imperfect inspection process.
- Comparative analysis of vendor-buyer collaborative integrated model vs. the buyer's independent decision model.
- Perform Sensitivity Analysis of cost parameters and their impact on total expected cost of inventories management.

Followings are scope of the research

- The research is for single vendor and single buyer.
- Only non-perishable items have been considered for this research work.
- The demand rate, production rate, percentage of defective items in production lot, inspection rate, type I and type II inspection errors are deterministic and known probability distribution.
- This research focuses on imperfect production quality items with imperfect inspection process.

The research work in this thesis covers the case of imperfect production quality and imperfect inspection process where the inspection process conducted by the vendor. It is a change in assumption from the earlier research works where researchers' assumed that the inspection process was conducted by the buyer. Further, it is assumed that the rate of inspection is greater than the production of items, so that the inspection process also finishes immediately after the end of the production of items, resulting in no extra delay due to the inspection process. As the inspection process is also assumed to be imperfect, it may wrongly classify non-defective items as defective (type I inspection error) and defective items as non-defective (type II inspection error). Because of type II inspection error, some defective items, classified as non-defective item could be sold in the market. After detection of defects, the consumer returns back (sales return) defective items to the buyer (here the dealer) and hence replaced with a non-defective item. Considering these assumptions following models have been developed. These models are tested and compared with the help of the numerical example using same numerical values that had been consistently used by earlier related inventory management research works.

- Integrated model where backorder has not allowed
- The Buyers independent decision where backorder has not allowed
- Integrated model where backorder has allowed
- The Buyers independent decision where backorder has allowed

Using numerical example, the minimum Expected Total Cost (ETC) of Integrated model where backorder has not allowed is 2,01,226.23$ which is lower than (Hsu & Hsu, 2012) model. For the model, where the buyers take independent decision and backorder is not allowed, the minimum ETC came to 2,08,459.45$, higher than the integrated model. It suggests that integrated model is better for reduction of minimum ETC.

With backorder being allowed in the inventory, the minimum ETC for integrated model is 2,00,516.0609$. For the model, where the buyers take independent decision and backorder is allowed, the minimum ETC came to

2,11,694.62$. This is higher than the integrated model. It is also higher than where backorder is not allowed. It suggests, that allowing backorder for the buyer independent model is not a good choice.

Sensitivity analysis shows that higher probability of defective items in production and higher rate of inspection error increasing minimum ETC very fast rate. Training of inspectors, use of technology and advanced equipment may reduce the inspection cost and reduce probability of type I and type II errors. The impact of quality management and training of inspectors could be taken as a future scope.

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LIST OF ABBREVIATIONS

CDF	Cumulative Distribution Function
CRM	Customer Relationship Management
ELS	Economic Lot Size
EOQ	Economic Order Quantity
ERP	Enterprise Resource Planning
ETC	Expected Total Cost
ETPU	Profit Per Unit Time
FEPQ	Fuzzy Economic Production Quantity
JELS	Joint Economic Lot Size
JIT	Just in Time
JTRC	Joint Total Relevant Cost
LTL	Less Truck Load
PD	Probability Distribution
PDF	Probability Density Function
PM	Preventive Maintenance
PMF	Probability Mass Function
QC	Quality Control
RFID	Radio Frequency Identification
SCM	Supply Chain Management
TC	Total Cost
TL	Truck Load
TRC	Total Relevant Cost

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CHAPTER 1

INTRODUCTION

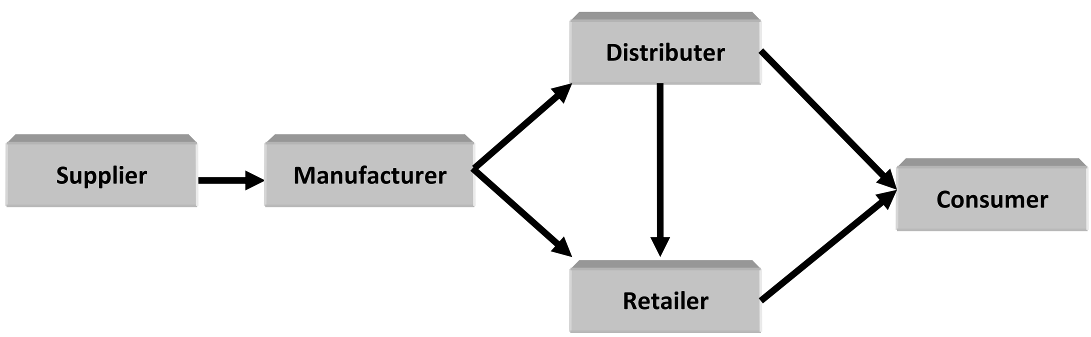
1.1 Concept of Supply Chain Management

The supply chain is a network of independent organizations connected together that performs procurement of raw materials, manufacturing of intermediate or finished products and distribution of finished products to its consumers by its distribution chain. The objective of the supply chain is to fulfill consumers' needs. Suppliers, manufacturers, transporters, warehouses, distributors, retailers and consumers since they too are stakeholders of the supply chain.

Supply Chain Management has an objective to efficiently integrate suppliers, manufacturers, warehouses/distributors, retailers and consumers' demand, so that the products are produced in the right quantities and distributed to the right locations at the right time to meet consumer demand with high quality services, (David et al., 2000). According to (Lee & Billington, 1993), the term "Supply Chain" gives an image in which a product or supply is moving from suppliers to manufactures to distributors to retailers and finally to consumer along a chain. Material flow, Information flow, Finance flow and Commercial flow are found in the supply chain. Material flow is a unidirectional flow. It starts as a movement of raw material from a supplier in the chain and finished product in the hand of the consumer. Information flow is bidirectional flow and consists of demand information flow, forecasting information flow, production and scheduling information flow, design and new product introduction information flow. Finance flow generated from the consumers and goes to all others in the chain. Buying and selling activities changes ownership of material from a party to another party. The change of ownership is referred as commercial flow. Material flow within an organization does not generate commercial flow. In reality, a manufacturer gets supply of raw materials from suppliers and supplies the finished products to distributors. Some cases the manufacturer supplied the finished products directly to retailers. The manufacturer provides guarantee on its product. Replacing defective items generated material flow in reverse direction. Therefore, a supply chain is a network, (Lee & Billington, 1993). These supply chain stages are given in Figure 1.1, and include the following:

- Customer
- Retailer
- Wholesalers/distributor
- Manufacturer
- Supplier (Raw Material)

Figure 1.1 The structure of the supply chain



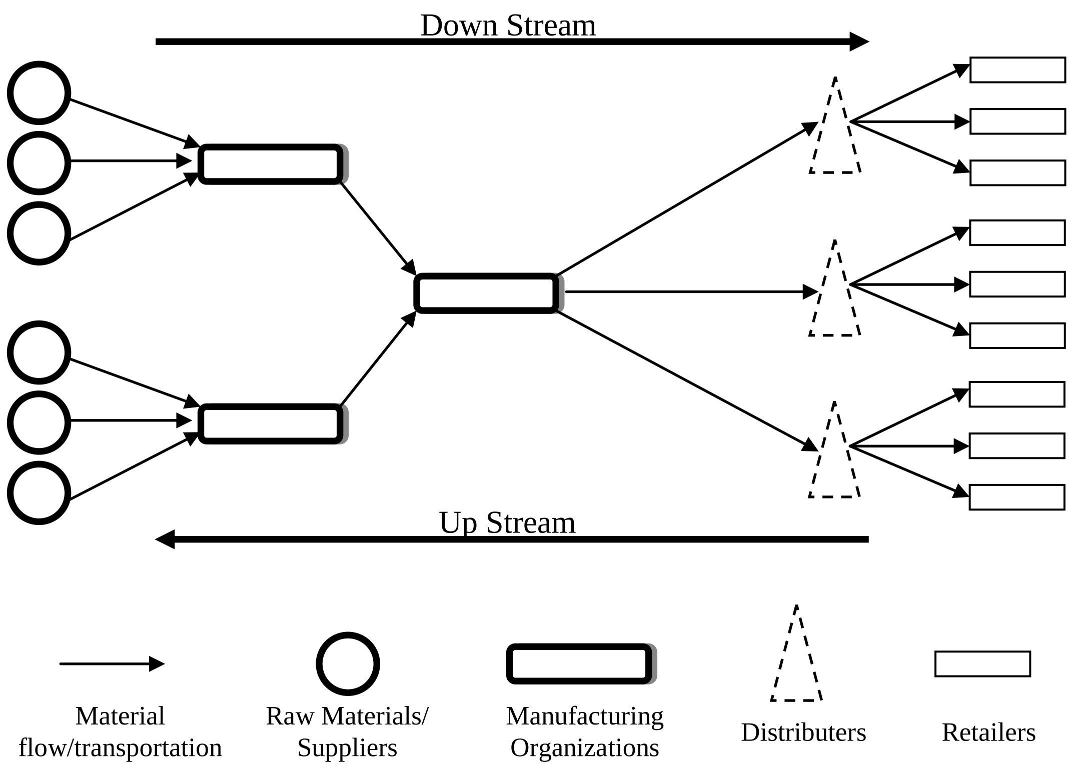
(Source: (Mandal, 2013))

Supply chain represents a network of facilities and distribution that procure raw materials, transform procured materials into intermediate and finished products and distributes intermediate or finished products to its consumers, (Chang et al., 2006). The supply chain is used in both manufacturing as well as in service sector organizations. The complexity of the supply chain depends upon industry to industry and organization to organization, (Agrawal & Raju, 1996).

Supply chain management actively manages supply chain activities to provide maximum values of products or services to its customers in sustainable competitive advantages, (Chen & Sarker 2010). Supply chain represents vigilant efforts of its each stage that has been developed in such a way that it runs supply chain in more efficient and effective manner. Its actives cover everything from raw material procurement, product design and development, sourcing, manufacturing, logistics and information sharing and management that is required for these activities, (Chen & Wang, 1996).

Organizations that are part of the supply chain management network are linked together through material flows, information flows, finance flows and commercial flows. The flow, which starts from the supplier and reaches to the consumers is also known as downstream flow. Material Flows and commercial flow are referred as downstream flow. The flow, which is initiated from consumer and reaches up to the manufacturer and some time to supplier is referred as upstream flow. Different types of Finance flows and Material flows include transformation, value addition of products and services, movement of products by logistic and management of inventories. The Material flow starts from suppliers of raw material and finally reaches to the end-user.

Figure 1.2 Supply Chain Network



(Source: (Mandal, 2013))

1.1.1 Supply Chain Decision

Supply chain Decisions have been classified into two major categories – strategic and operational. Strategic supply chain decisions are taken for a longer period of time. These are very closely associated with the cooperating strategy of organization, and guide supply chain policies for its design perspective. Whereas operational decisions are for short periods of time and its focus on the day to day activities of supply chain management. The focus of these decisions is to effectively and efficiently manage the different types of flows in the strategically planed supply chain.

Supply chain management has four major decision areas (Vonderembse, 2006):

1) Location
2) Production
3) Inventory
4) Transportation

Efforts are made to successfully carry out strategic and operational decisions in these areas. Out of above four decision areas, the inventory carrying cost could be reduced by correct operational decisions.

1.1.2 Inventory Decision

Inventory decision refers as to how inventories are managed. Inventories exist at each level of the supply chain. It can be for either raw-material, semi-finished product or finished product. Most of the approaches for the management of inventories are in an operational perspective. It includes deployment strategies (push or pull), control policies, determination of optimal order quantities, fixing the safety stock level, and finding reorder points for each inventory location of the supply chain. (Cachon, 2004)

1.2 Principles of recent Supply Chain Model

Following principles are used to describe the recent supply chain model:

End customer of a supply chain is the entity that puts money in the supply chain (Chen & Sarkar, 2010). Only that solution is stable in which every element of the supply chain, from the raw material supplier to end consumer gets profit from the business (Chang et al., 2006). Supply chain management is about the value addition of products, the total content of a product and service (Yoo et al., 2009). There are three major pillars of recent model: managing the supply chain, material flow and managing information.

As the global marketplace becomes more and more competitive, coordination in supply chain management becomes a key component for improving its profitability and responsiveness. With no coordination among supply chain members, supply chain members work independently and try to maximize their own profit, which may not result in an optimal solution for the whole supply chain from economical and environment points of view, (Sajadieh & Jokar, 2009). With the help of coordination between the vendor and the buyer, total profit of the supply chain is maximized and the loss-making member in the chain is usually compensated in a way so that everyone gets its benefits.

To achieve an effective coordination between a manufacturer and consumer of its product will always remain a concern for management and a challenge for researchers. Academicians and practitioners have been given an increasing attention to vendor-customer coordination problem, (Jaber & Zolfaghari, 2008), (Ben et al., 2008). Customer-vendor coordination problem is being referred as the Joint Economic Lot Sizing (JELS) problem with stream of research works. The traditional inventory management used two-echelon supply chain inventory and shipment policy for the vendor and the customer that is managed independently by the vendor and the customer due to that the optimal size for the customer may not result in an optimal policy for the customer, and vice versa.

(Goyal, 1976) had first time introduced 'Joint Total Cost' for an integrated single-vendor, single-customer system where production rate was infinite for the vendor and shipment policy was 'lot-for-lot'. (Banerjee, 1986), had worked further with finite rate of production. (Goyal, 1988) relaxed 'lot-to-lot' shipment policy to a number of equal size lots shipment policy. These models deal with optimization of integrated inventory for a vender-customer system and shipment policy, minimizing the total inventory cost with an assumption that both the vendor and the buyer are cooperating with each other to reduce of total inventory cost.

The JELS model proposed by (Goyal, 1977), (Banerjee, 1986) and (Goyal, 1988) had been extended in many directions in future research. These could be divided into categories of “Quality”, “Controllable lead times” and “Transportation”.

Excellence, meeting and exceeding consumers’ expectations are key objectives of quality management. These objectives lead to a product standard specification. Quality management activities make sure that all products manufactured have no deviation from the standard specification of the product. In practice, there are some deviations in products, so quality management tries to keep these deviations to a minimum by a series of quality checks. In supply chain management, production as well as inspection process of finished goods is treated as imperfect.

1.3 Inventory Control

“Inventory is the stock of goods held for doing business”, (Kelebu, 2013). Inventory management expense, which is typically 45 to 90 percent of all expenses. It ensures that a sufficient number of products are in the inventory, to avoid the product to be out of stock, to prevent spoilage or theft, to have a proper accounting of products. Inventory control involves the accounting of products, maintaining the correct label of product/material, giving orders in the optimal number of quantities, care and accounting inside the inventory, and finally disposal of product/material. Following are the reasons for inventory control:

- Management of inventory locations and their storage
- Management of supply of products from different inventories to receivers

- Keeping records of inventory issued for receivers
- Provide prompt and proper services to all concerned
- Maintain inventory at the lowest cost
- Bifurcate high-value and low value products
- Avoid over-stocking and under-stocking of products/raw materials
- Some products have an expiry date. It maintains the record of expiry dates and issues older stock first such that product are sold well before reaching their expiry date.

1.4 Types of inventories on the basis of their use

1.4.1 Decoupling Inventory

The manufacturing process has a successive series of operations, making a chain of operations. Each operation in the chain does some specialized job. (Pulat & Pulat, 1992) An operation receives raw material / semi-finished product from the previous operation; performs its job on product and passes the semi-finished product to the next operation in the chain. This process continues until the finished product is manufactured. Each operation in the chain is adding some components to products depending on the job they do. Inventories of components are maintained to make a smooth supply to each operation point. In the production process, even a small cache inventories are being maintain for each component near their operation points and get replenished frequently from their inventories. Any interruption or delay at any stage in the chain could adversely affect the entire production process. This type of inventories is referred as decoupling Inventory. On the basis of above inventory is classified as

- Raw material and components
- Work in process inventory
- Spare parts inventory
- Consumable items, such as lubricant, stationary, cleaner and many other items used by manufacturers in day-to-day operation.
- Finished product inventory (available for sale)

1.4.2 Inventory Lot Size

Production of the products is done in batches. Products produced are stored in inventory at the production site. A certain number of products in a lot, which is very less than the production batch, are shipped to the buyer and the remaining products are left in the inventory for future shipments.

1.4.3 Safety-Buffer Stock

This stock is maintained to deal with uncertainty in demand such as certain high demand in the market, or in emergency situations where fresh shipment gets delayed.

1.4.4 Pipeline Inventory

The stock that is in transit from the manufactures to the buyer and expected to arrive to the buyer's inventory at any time is referred as pipeline inventory.

1.4.5 Seasonal Inventory

Demand of product may be not same all the time in a year. During some part of the year demand can be higher and in some part of the year can be low. The inventory that is designed to meet high demand during a time interval of a year (season) is seasonal inventory.

1.4.6 Anticipation Inventory

In market, due to some special circumstances, demand of any type of product may go up all of a sudden. When the buyer, anticipates the surge in demand in advance, try to arrange extra inventory of products to meet additional demand. This type of additional inventory is called Anticipation Inventory.

1.5 Basic Concepts and terminology

The following are the basic terminology used in inventory system

Demand – Demand is defined as the number of units that are required for a given time in an interval. Based on the past pattern and experience, demand of current time is hereby, defined. There are two types of demands, 'Deterministic Demand' and 'Probabilistic Demand'. When based on previous years' demand pattern, the demand in a given time duration can be known in advance with certainty is said to be 'Deterministic Demand'. When the demand for the given time duration could not be known in advance with certainty and may be predicted by some probability distribution using past years' data is said to be 'Probabilistic Demand'.

Lead Time – Lead time is the time gap between the initiation of the procurement process and the actual receipt of the order. It has two components.

- Administrative lead time
- Delivery lead time.

Administrative lead time is the time duration that is taken by the administration from initiation of the procurement process to placing its final order. Delivery lead time is the time duration that is taken between placing an order to getting actual delivery of ordered material. It can be deterministic or probabilistic.

Planning Horizon – This is the time interval for which a particular inventory level has been maintained. It can be finite or infinite depending upon the demand of materials.

EOQ (Economic Order Quantity) – Total inventory cost depends upon the number of items ordered. The order quantity that gives a minimum overall cost is known as EOQ.

Order Cycle – Order Cycle involves a process that starts from initiation of procurement process and end with receipt of ordered item shipment. The process is repeated in supply chain management. Quantities of items in a fresh order depend upon following two types of inventory review.

1.6 Description of different terms used to determine total cost and profit of inventory system; in brief

Setup Cost – Production unit needs to get prepared before start of production for one type of product. Each preparation of production unit needs certain types of costs like cost for making arrangements, loss of production during configuration etc. This cost is referred as setup cost.

Production Cost – Each unit produced required raw material, labour, electricity and much more. These are available at some cost and each unit produced has per unit production cost.

Inventory Carrying (Holding) Cost – Many types of inventories are maintained in the supply chain to make sure the item or material is always available. To operate and maintain an inventory some costs are involved because of storage space, storage facilities, handling equipment, operational equipment, skilled/unskilled labour, security, insurance, interest on the money invested and other expenses.

Ordering Cost – This cost is associated with ordering fresh items (raw material or finished item). It includes a publication notice in various media, gathering quotation, stationary used, postage, telephone and internet charge etc.

Purchase Cost – This is the unit cost of an item that is paid by the buyer to the vendor. It includes the production cost of the item plus the profit of the vendor.

Shortage Cost – When an item is out of stock, some shortage cost occurs. It consists of loss of profit, loss of brand value, loss of goodwill and many other indirect costs involved.

Discounting Rate – It is an interest rate that is used to calculate present value of future cash flow.

1.7 Different types of Inventory Models

1.7.1 Deterministic Inventory model

Basic Economic Order Quantity (EOQ) was given by (Harris, 1913). His work was later published by Donald Erlenkotter in Operation Research journal in the year 1990. The EOQ is calculated for single echelon, single item with deterministic lead time and constant rate of deterministic demand without shortage for buyer inventory. The formula for EOQ is given by

$$Q*(EOQ) = \sqrt{\frac{2dK}{h}} \tag{1.1}$$

Where	d	:	Annual demand
	K	:	Ordering cost
	h	:	Holding/Carrying cost

This EOQ model is the basic model and it becomes the base for the development of all other deterministic models. This model was used in first and second world wars for optimization of inventory. During and after the Second World War different optimization technique was developed and research on the inventory model had gained momentum. After publication of deterministic models by (Goyal, 1977), (Banerjee, 1986) and (Goyal, 1987) the research on the inventory model had gained further momentum and it got diversified in following types of inventory models:

i) Constant rate of demand and variable order cycle

ii) Constant rate of demand and fixed reorder cycle time

iii) Gradual supply, allowing shortage

It further diversified by considering different types of constraints like warehouse space, investment, average inventory level, number of orders, quantity discount etc. JIT (Just in Time) concept was developed and used by advanced countries like Japan where inventory level has been reduced to almost zero by synchronizing and reducing lead times to a minimum.

In recent years, research on the inventory model has further diversified to stochastic and fuzzy models. Research in the area of deterministic inventory is still in progress, making these models more practical for manufacturing and marketing systems. (Benkherouf, 1995) had given a method to find an optimal replenishment schedule with a shortage of inventory, item-level reduced at a constant rate with known reducing demand.

1.7.2 Probabilistic Inventory model

In the deterministic inventory models demand and lead-time are assumed to be known exactly based on past records and experience, but in probabilistic inventory models demand and lead-time are not known exactly. Different type of probabilistic models has been developed for different situations. Following is the list of types of probabilistic models

- Single period models (static demand model)
- Multi period models (variable lead-time with dynamic demand)

Single period models – In this type of models items required are as per the demand and ordered in one single lot (period).

Multi period models – In this type of models, inventory level of items are being regularly reviewed. Based on the review method, order for items is placed. There are two types of review methods.

- *Periodic Review* – Inventory levels of items are reviewed at constant time interval. Ordered quantity for each item is the difference between their respective highest inventory levels to their current inventory levels. After receiving the ordered items, inventory level of each item reached back to their highest inventory level.
- *Continuous Review* – Inventory level are continuously being reviewed. As soon as inventory level of any item reaches below its reorder level, a fixed number of quantities are ordered. Recent days, continuous review is done with help of computer applications. The computer application records all the incoming and outgoing items in the inventory and gives an up-to-date inventory level of each item. When level of any item fall below reorder level, it gives trigger for order to be placed.

As per (Hillier, 2012), demand D in the probabilistic inventory system is not known exactly and it is a random variable with known probability distribution of D. Let

$$PD(d) = P\{D = d\} \qquad (1.2)$$

Where values of P_D (d) are known for d = 0, 1, 2, ...

Let S is the maximum level of an item and I items are already available in the inventory. Q items, (S – I) will be ordered to make inventory level back to S. Let K is ordering cost, c is cost unit of an item, his inventory holding cost for per unit item for unit time and b is shortage cost. The number of items sold in the market will be given by

$$\min(D,S) = \begin{cases} D & if\ D < S \\ S & if\ D \geq S \end{cases} \quad (1.3)$$

The cost is incurred

$$C(D, S) = K + c\ (S – I) + b \max\ \{0, D – S\} + h \max\{0, S – D\} \quad (1.4)$$

As demand is a random variable the above cost is also a random variable. Expected cost C(S) is

$$C(S) = E[C(D,S)] = \sum_{d=0}^{\infty}(K + c(S - I) + b \max\{0, d - S\} + h \max\{0, S - d\})\ P_D(d)$$

$$C(S) = K + c(S - I) + \sum_{d=S}^{\infty} b\ (d - S)P_D(d) + \sum_{d=0}^{s-1} h(S - d)P_D(d) \quad (1.5)$$

It is dependent on probability distribution of D. When demand ranges give a large number of possible values for discrete random variable it is difficult to find probability distribution. Continuous random variable is taken in place of discrete random variable.

Let *f(x)* = probability density function of D and

Cumulative Distribution Function (CDF) of D is

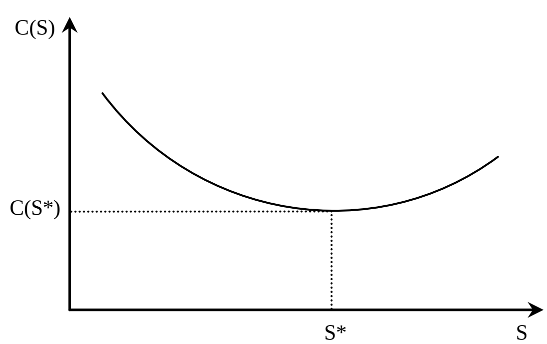
$$F(d) = \int_0^d f(x)dx \quad (1.6)$$

Expected cost C(S) for continuous random variable is

$$C(S) = E[C(D,S)] = K + c(S - I) + \int_S^{\infty} b\ (x - S)f(x)dx + \int_0^S h(S - x)f(x)dx \quad (1.7)$$

The second derivate of above is nonnegative for every values of x, therefore it is strictly convex function. C(S) has a global minimum S*. Following figure show the function

Figure 1.3
Strictly Convex function in inventory management



Source: (Hillier, 2012)

1.8 Objective and scope

Supply Chain Management is a very challenging field. Lots of researches have been done on it. These researches have tried to optimize total cost of supply chain and reduce the inventory carrying cost. Supply chain is itself a very complex system and operates in many different situations. Researchers had been trying to cover these situations in their research works. Companies working in supply chain management have been benefited by these research works. However, still there is a lot scope where further research work is required.

Objectives of this research are:

- Identify impact on total cost of the supply chain management when inspection is being performed at the vendor site for imperfect production quality and imperfect inspection process.
- Comparative analysis of vendor-buyer collaborative integrated model vs. the buyer's independent decision model.
- Perform Sensitivity Analysis of cost parameters and their impact on total expected cost of inventories management.

Scope of the Research

- The research is for development of mathematical model of single vendor and single buyer supply chain management where objective of the vendor and the buyer are to minimize inventories carrying cost.
- Only non-perishable items have been considered for this research work.

- The demand rate, production rate, percentage of defective items in production lot, inspection rate, type I and type II inspection errors are deterministic and known probability distribution.
- This research focuses on imperfect production quality items with imperfect inspection process.

1.9 Organization of the thesis

Chapters of the thesis are organized as described below:

Chapter 1

This is an introduction chapter. It explains the concepts of supply chain management. The relation between stakeholders in the supply chain structure has been discussed. There are different types of flows in supply chain management. These flows along with supply chain decision, inventory decision, types of inventory and the basic terminology used in the supply chain have been discussed in this chapter. The recent trend of supply chain model development in deterministic models and probabilistic models have been taken into account. The research objectives and overview of the structure of the thesis has been presented.

Chapter 2

This chapter covers literature review. A broad understanding is gained from the literature review. The review starts from the first solution provided by (Harris, 1913) and continued till the latter developments. Joint economic lot-sizing models and integrated vendor-buyer inventory models with imperfect quality are studied in detail.

Chapter 3

This chapter explains the methodology used in this thesis. It explains mathematical modeling. Steps of mathematical modeling, which are the pillars of this thesis, is explained in detail. To find optimal values from a mathematical function curve, the nature of the curve is checked. The strictly convex curve has a minimum value and a strictly concave curve has a maximum value. The chapter explains the condition for checking the convexity of a mathematical function. Theory of Exception and "Renewal and Reward Theorem" is explained.

Chapter 4

This chapter covers details of the development of models for single-vendor, single-buyer integrated inventory with imperfect production quality and imperfect inspection where the inspection is conducted at the vendor's site has been developed. A better solution was found out. The solution has been explained with the help of numerical illustration. Sensitivity analysis explains the impact of different parameters on the minimum expected total cost.

Chapter 5

In this chapter, the models developed in chapter 4 has been extended by allowing backorder. By allowing backorder, a better solution was found which has a better minimum expected total cost. The solution is explained with the help of a numerical illustration. Sensitivity analysis explains the impact of different parameters on minimum expected total cost.

Chapter 6

This chapter concludes the findings, implications of the research in the thesis. The result and sensitivity analysis have been discussed in detail. Based on findings and analysis, a few suggestions are also made. Limitations and scope for future work have been discussed.

CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

In this chapter Review of Literature is done in the field of integrated inventory of supply chain management and tries to get a better understanding. First and basic formulation of inventory was given by (Harris, 1913) in 1913. Though, his original work was not found in any literature, his work has been recognized and cited by many researchers. He had given the formula for single echelon system and considers only one place where the inventory has been maintained. His formula was used in the first world and second world wars for the management of inventories.

Industry and academia have worked together on inventory management in supply chain management and made a significant improvement in minimizing overall inventory carrying cost. After globalization, integration of systems and high competition, industries have more pressure to cut down inventory costs further to improve their performance and remain in the competition. Now they cannot operate as an individual, but have to cooperate with each other in the reduction of inventory cost. (Hadley & Whitin, 1963) had tried to give an answer to the basic questions of inventory management that when and how much order will be placed. The evaluation of information technology allows fast and efficient flow of information. This helps the industry to manage their inventory more efficiently.

The research in the area of inventory management of supply chain got triggered when (Goyal, 1976) had published his paper for inventory model having single-vendor, single-buyer for a single product with an unrestricted production rate and lot-for-lot policy.

(Goyal, 1977) had given an integrated inventory model and found a joint economic lot size for single-vendor, single-buyer where both the vendor and the buyer work together to minimize overall cost. This overall cost is not the minimum cost for either of the two. This model shows that if the buyer and the vendor operate individually and try to work for their individual minimum cost, the total of their individual costs was more than of integrated cost. An integrated inventory system for single-vendor and single-buyer is being considered as a basic building block for inventory models. This basic building block is used to answer complex inventory models.

(Banergee, 1986) replaced infinite production rate of (Goyal, 1977) with finite production rates at the vendor and give the more generalized model.

(Goyal, 1988) further generalized the work of (Banerjee, 1986) by removing lot-for-lot policy and assumed that after receiving the purchaser's order the vendor produced all items as per the order. After completion of production, the vendor shipped equal size lots to the purchaser which is integer multiple of purchaser's order.

Lee & (Rosenblatt, 1987) had considered a machine failure, machine maintenances and restoration, and length of production run during the production process. They developed a relationship to determine effectiveness of maintenance by inspection and showed that optimal inspection intervals are equal. They provided optimal economic order quantity (EOQ) and inspection schedule.

(Lu, 1995) relaxed (Goyal, 1988) assumption and provided solutions where the vendor started shipping items to the purchaser before the production of all items has been completed.

(Goyal, 1995), relaxed the assumption that all lots of equal size and proposed more generalized assumption that ratio between two consecutive lots will be equal.

It is a general practice used by suppliers to give some discount to his/her present customers in such a way that they have to purchase more items. Finally leading to decrease in profit, but the increase in cash flow translated into an increase in overall profit for the supplier. (Monahan, 1984), analysed, how a supplier could encourage his major customers to increase the present order quantity by a factor of “K”, by providing a structured term for quantity discount schedule for one-item, one-vendor and one-customer. He had found an optimal level for “K” and price discount given in order so that the supplier gets maximum profit. (Joglekar, 1988), through his note questioned that Monahan’s one-item, one-vendor and one-customer model is not reasonable and he also raised that inventory carrying of unsold items, as production of items is bigger than ordered items, should also be taken in consideration.

(Wagner & Berman, 1995) had given a stochastic model for planning capacity expansion for convenience store chains that take care of uncertainty of future demand and remain within the budget constraint and other resources. The model has provided size, location and timing of expansion to maximizing expected profit.

(Banerjee & Kim, 1995) had discussed integrated just-in-time (JIT) model. The buyer needed Q items at a regular interval. For the vendor it had been not economical to produce Q items at a regular interval as overall cost could be high due to frequent production set-up. It would be economical to produce NQ items in one production setup and sent items in economical lots sizes such that both parties would get benefitted.

(Viswanathan, 1998) discussed about two replenishment strategies in integrated vendor-buyer inventory model. In the first strategy, the vendor replenished the buyer with equal quantity in each delivery. In the second strategy, the vendor sent all available items from his inventory to the buyer in each delivery. He analysed both the strategies and had found that none of the strategy is provided the best result in all the possible parameters.

The life span of perishable items in the market is very limited. If it is not sold at a fixed time span, the net value becomes scrap. Newspaper falls in this category. If it is not sold on the day, then it became scrap next day. Under order and over order both leads to loss in profit. (Weng, 2004) had given a newsvendor model. The model analyses and helps the manufacture and the buyer in a coordinated manner for taking a decision on number of Newspapers to be printed. The model also discusses on the quantity discount as the incentive policy so that there is a coordination of the buyers order on the quantity.

2.2 Joint economic lot sizing models

Joint economic lot sizing models is generally two echelon systems consist of one vendor and one or multiple buyers based on deterministic joint EOQ. The cost function in two echelons is sum costs incurred by both parties in the management of inventories. By research, it has been established that the overall cost in integrated system is less than if the cost for the vendor and the buyer are optimized separately. (Goyal, 1977) was the first to analyse integrated vendor-buyer model with infinite production rate. He stated in his work that the price and size of lots of items were decided after negotiation between the purchaser and the vendor. Both parties looked for their own benefits and their negotiation resulted in near optimal or optimal for any one party and sometimes non-optimal for both parties. (Banergee, 1986) modified (Goyal’s, 1977) model with finite production rate and lot-to-lot policy. In this policy, after receiving an order from the purchaser the vendor starts producing ordered items. When all items are produced, the vendor sends all produced items to the buyer. (Goyal, 1988), through his model, removed lot-to-lot policy. (Goyal, 1995) generalized integrated model by relaxing equal size lots and proposed a ratio between the two consecutive lots. When the ratio is 1:1 then it deals with equal size lots otherwise it may be increasing or decreasing as per the ratio. (Lu, 1995) further extended (Goyal, 1995) model and provided a solution where shipment of items did not wait till the completion of production. As soon as items required for shipment of a lot have produced, lots of items are shipped to the buyer. (Hill, 1998) generalized more, the integrated model and proposed that the lot size is increased by a factor within a production batch. (Hill & Omar, 2006) re-examined integrated models at that time. He had discussed that unit holding cost is not fixed. He said the assumption, the unit holding costs will increase as inventory level goes down in the supply chain, which is not always true. He said that lot size of each shipment should not be equal. He had given an optimal model considering the parameters of a fixed production set up cost, fixed ordering cost, holding cost for the vendor and for the buyer and fixed cost for each delivery.

(Barron, 2001) had developed a new approach to EOQ with backlogging using algebraic approach. He had proposed extension of Grubbstrom & Erdem's algebraic procedure to find optimal values.

(Banerjee, 2005) had dealt with 'Production Environment' where supplier produced items as per 'make-to-order' contract and simultaneously determined sales as well as lot size. The objective of supplier is to set price of a unit to get targeted gross profit per unit.

(Banerjee, 2009) discussed transport economics in supply chain management. He stated that a full truckload (TL) had been more economical than that of less truck load (LTL) shipments. He had re-examined the economic lot scheduling problem later developed an integrated production schedule for multiple items as per a shipment plan that could be shipped in full truckload and could achieve economic lots along with transport economics.

(Bylka, 2009) tries to minimize individual average total cost of production, shipment and inventory holding for non-cooperative strategy in a restricted non-cooperative game. Through this research work, the author tries to explain that non-cooperation could also be a feasible strategy for getting optimal costs for vendor-buyer.

(Barrón et al., 2011) had mentioned that National Semiconductor, Wal-Mart, and Procter and Gamble was successful using the integrated supply chain for the vendor-buyer system that in-turn helped them to reduce their joint inventory cost, response-time and improving their performance and market share. Using arithmetic-geometric inequality, he had proposed an alternative approach to determine global optimal inventory policy for integrated vendor-buyer system.

2.3 Integrated vendor-buyer inventory models with imperfect quality

(Salameh & Jaber, 2000) were first to point out assumptions made in supply chain management inventory modes, which were not realistic and they assumed that items received by the buyer contain defective items with a known probability density function. They proposed a 100% screening of received items. Poor quality items are stored in inventory and sold in market at a discount rate, just before receiving shipment of next lot. They showed that optimal economic lot size increases with an increase in average percent of imperfect items increase.

(Wee et al., 2007) had developed an optimal inventory model for imperfect items with backorder. They suggested that removal of poor quality items from the stock result into a shortage of items and over production of items result into an increase of overall inventory cost. They rejected above-mentioned solutions and allowed backorder in their model. A 100% screening of items conducted at the buyer site after receiving fresh items' lot.

(Maddah & Jaber, 2008) founded a flaw in random fraction of imperfect quality items and a screening process in (Wee et al., 2007) and corrected the model.

(Khan et al., 2011) mentioned that (Salameh & Jaber, 2000) had not taken care about errors committed by inspectors during 100% screening process and they mentioned that inspectors may commit two types of errors. Type I error and type II error. They assume that probability of error in production and screening process are known from historical records.

(Hsu & Hsu, 2012b), pointed out a flow in (Wee et al., 2007) model that all backorder immediately got cleared up with arrival of new items' lot. (Wee et al., 2007) did not consider the fact that screening of items would take some time and clearance of backorder would take some time. (Hsu & Hsu, 2012b) by a technical note corrected the flow and gave a corrected model.

(Hsu & Hsu, 2012a) had extended (Khan et al., 2011) model. (Khan et al., 2011) had assumed production lot is equal to order lot. (Hsu & Hsu, 2012) changed and assumed that production lot is multiple of ordered lots.

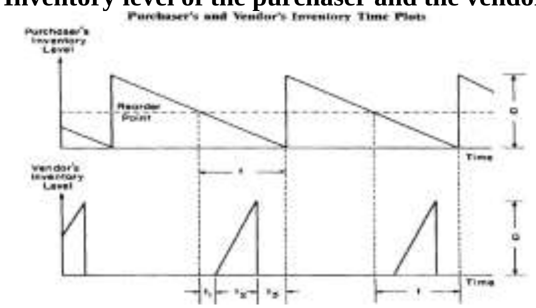
2.4 Table 2.1 Summary of relevant literature review of research articles relevant for this study

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
1	An integrated inventory model for a single supplier single customer problem.	Research Paper *International Journal of Production Research*	Goyal, S. K.	1977	(Goyal, 1977) developed integrated model for single customer and single supplier and compared results of integrated solution with individual solutions. This model was first integrated model. **Figure 2.1** **Costs of integrated model** $\text{Cost to the supplier} = Z \cdot V(C(t^*), S(t^* \cdot K(t^*)))$ $\text{Cost to the customer} = (1-Z)V(C(t^*), S(t^* \cdot K(t^*)))$ $\text{where } Z = \frac{V(S(t_0 \cdot K(t_0)))}{V(S(t_0 \cdot K(t_0))) + V(C(t_0))}$ Source: (Goyal, 1977) Where for customer => D = demand per unit of time, R = Cost of purchase order, h1 = customer stock holding cost per unit per unit time, t = time interval between successive order, V(C(t)) = Variable cost unit of time, for supplier => M = Setup cost, h2 = suppler stock holding cost per unit per unit time, T = time interval between successive set-up, K = T/t a positive integer, V(S(tK)) = variable cost per unit of time.	(Goyal, 1977) models was simple model and did not consider rate of production.
2	A joint economic-lot-size model for purchaser and vendor.	Research Paper *Decision sciences*	Banerjee, A.	1986	(Banerjee, 1986) developed Joint Economic-Lot-Size (JELS) and Joint Total Relevant cost (JTRC) for supply chain management with the assumption that the vendor produce products as per the order received from the purchaser on a lot-for-lot basis under deterministic condition, (Banerjee, 1986). Figure 2.2 shows inventory levels of the purchaser and the vendor over time. At reorder point the purchaser placed order,	It is not realistic for the vendor to produce item lot-for-lot each time after receiving an order from the purchaser. The vendor may

<table>
<tr><th>Srl. No</th><th>Title of paper</th><th>Literature type</th><th>Author</th><th>Publishing Year</th><th>Contribution</th><th>Gap / Future work</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>

after t_1 time production started by the vendor which continue for t_2 time. After completion of production the lot of items shipped to the purchaser which reached to the purchaser in t_3 time. Thus $t = t_1 + t_2 + t_3$.

Figure 2.2
Inventory level of the purchaser and the vendor



Source: (Banerjee, 1986)

The formulation of costs for the purchaser and the vendor is shown in the Figure 2.3.

Figure 2.3
Summary of costs for individual optimal policies

Summary of Relevant Costs and Individual Optimal Policies

	Purchaser		Vendor	
General cost function	$TRC_p(Q) = \frac{DA}{Q} + \frac{Q}{2} rC_p$	(1)	$TRC_v(Q) = \frac{DS}{Q} + \frac{DQ}{2P} rC_v$	(4)
Economic lot size	$Q_p^* = \sqrt{[\frac{2DA}{rC_p}]}$	(2)	$Q_v^* = \sqrt{[\frac{2PS}{rC_v}]}$	(5)
Minimum total cost	$TRC_p(Q_p^*) = \sqrt{[2DArC_p]}$	(3)	$TRC_v(Q_v^*) = D\sqrt{[2SrC_v/P]}$	(6)

Note: $TRC_p(Q)$ = purchaser's annual total relevant cost for any lot size Q, $TRC_v(Q)$ = vendor's annual total relevant cost for any lot size Q, Q_p^* = purchaser's economic lot size (ELS), Q_v^* = vendor's economic lot size (ELS).

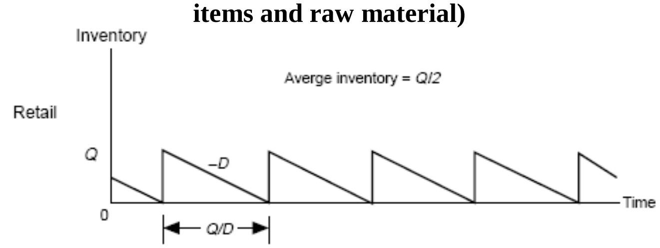
Source: (Banerjee, 1986)

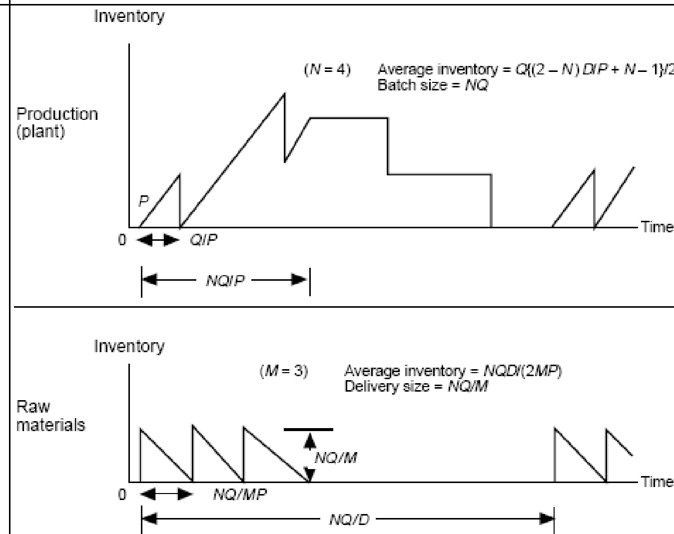
</td><td>produce more items and send items in future. It is simple model and did not consider realistic assumptions like imperfect production quality, stochastic inventory etc.</td></tr>
</table>

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
					The formulation of costs for the joint policy is shown in the Figure 2.4. **Figure 2.4** **JTRC and JELS – Q_i*.** $JTRC(Q) = {}^{D}/_{Q}(S+A) + {}^{Q}/_{2}r({}^{D}/_{P}C_v + C_p).$ $Q_J^* = \sqrt{[\frac{2D(S+A)}{r({}^{D}/_{P}C_v + C_p)}]}.$ Source: (Banerjee, 1986) Where D = annual demand, S = setup cost for the vendor, A = ordering cost per order for the purchaser, r = annual inventory carrying charges, C_v = unit production cost occurred to the vendor, C_p = unit purchase cost to the purchaser and Q = order or production lot size in units	
3	Determination of Production Cycle and Inspection Schedules in a Production System.	Research Paper *Management Science*	Lee, H. L. & Rosenblatt, M. J.	1987	(Lee & Rosenblatt, 1987) considered production of single item on single machine where at the beginning of production, the production process is in an "in-control" state. It produced perfect quality items with negligible number of defective items. As time goes the production process deteriorates and shifted to "out-of-control" state and produced defective and sub-standard items. (Lee & Rosenblatt, 1987) assumed that the production process remain in "in-control" state for a random time duration which is exponentially distributed with mean 1/µ. (Lee & Rosenblatt, 1987) assumed that inspections of the production process were carried at end of each production run.	During production process machine goes for wear and tear and it impact on the quality of item produced. (Lee & Rosenblatt, 1987) had tried to find impact of the wear and tear on production. (Lee & Rosenblatt,

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
					If the production process was found in "out-of-control" state, a restoration work was carried out at some cost. At start of each production cycle, production is in "in-control" state. Using above assumptions (Lee & Rosenblatt, 1987) tried to derive Economic Manufacturing Quantity for a production cycle and inspection schedule. **Figure 2.5** **The optimal production run duration T for n inspections per run** $$T^*(n) = \left[\frac{2(K+nv)D}{P(P-D)h + D(s\alpha P/\mu - r)\mu^2/n}\right]^{1/2}$$ Source: (Lee & Rosenblatt, 1987) Where D = Demand rate, P = Production rate, T = Cycle time for production lot, K = Setup cost, s = cost incurred by producing a defective item (rework, repair, replacement, loss of goodwill, etc.), α = percentage of defective units, υ = cost of inspecting the production process, r = cost of restoring the production process, n = number of inspections per production run, n ≥1, T_i = elapse time from beginning of production run until the i^{th} inspection.	1987) discussed production of imperfect quality items during the production. They did not focus much on impact on supply chain.
4	A joint economic-lot-size model for purchaser and vendor: a Comment	Research Paper *Decision sciences*	Goyal, S. K.	1988	(Goyal, 1988) generalized (Banergee, 1968) model by removing lot-for-lot policy, assumed that the vendor may produce an integer multiple of order lot quantity and supply multiple lots from a production run. The economic order quantity (EOQ) obtained by him is shown in the figure 2.6	(Goyal,1988) proposed multi shipment policy in inventory management and showed that it was better than lot-to-lot

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
					Figure 2.6 **Economic Order Quantity (EOQ) for the purchaser** $$Q(n)=\left(\frac{2D(A+\frac{S}{n})}{r(C_Q-C_v+nC_v(1+\frac{D}{P}))}\right)^{1/2}$$ Source: (Goyal, 1988) He calculated joint total relevant cost (JTRC) for the vendor and the purchaser. The JTRC is given in figure 2.7 **Figure 2.7** **Joint Total Relevant Cost (JTRC)** $$JTRC(n)=[2Dr(A+\frac{S}{n})(C_Q-C_v+nC_v(1+\frac{D}{P}))]^{1/2}$$ Source: (Goyal, 1988) The optimal value of n* is calculated using the condition given in the figure 2.8 **Figure 2.8** **The optimal value condition for n*** $$n^*(n^*+1)\geq\frac{S(C_Q-C_v)}{AC_v(1+\frac{D}{P})}\geq n^*(n^*-1)$$ Source: (Goyal, 1988) Where D = annual demand, S = setup cost for the vendor, A = ordering cost per order for the purchaser, r = annual inventory carrying charges, C_v = unit production cost occurred to the vendor, C_Q = unit purchase cost to the purchaser, Q = order or production lot size in units and n = multiple of order such that production quantity = nQ.	production policy. This model becomes basic model for future models

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
5	An integrated JIT inventory model	Research Paper *International Journal of Operation & Production management*	Banerjee, A. & Kim, S. L	1995	As per (Banerjee & Kim, 1995) in the Just in Time production system a buyer may order some fixed quantity Q at a regular interval of time. They pointed out that if the vendor (manufacturer) produced ordered items for a lot-for-lot policy then there would be production set-ups for each Q quantity produced, which is very frequent. They suggested it would be more economical that the vendor will produce NQ items in one production lot and send N number of Q items to the vendor and the same time raw material supplier also supply raw materials at regular intervals. The inventory levels for the buyer, the vendor (finished items and raw material) is explained in the figure 2.9 **Figure 2.9** **The inventory levels for the buyer, the vendor (finished items and raw material)**  	(Banerjee & Kim, 1995) had discussed lot-to-lot production for JIT and showed inventory levels for the vendor and the buyer.

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
					 Source: (Banerjee & Kim, 1995) As per (Banerjee & Kim, 1995) model optimal Q* in Just-In-Time could be obtained formulas given in figure 2.10 **Figure 2.10** **Calculation of Q* in JIT** $Q^* = (\alpha / \beta)^{\frac{1}{2}}$ where $\alpha = 2D[(A_m M + S)/N + A_r]$ and $\beta = Nh_m D/(MP) + h_p[(2 - N)D/P + N - 1] + h_r$ Source: (Banerjee & Kim, 1995) With optimal condition for M* (figure 2.11) and condition for N* (figure 2.12)	

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
					Figure 2.11 **Condition for M* in JIT** $M^*(M^* - 1) \leq \delta/\phi \leq M^*(M^* + 1)$ Source: (Banerjee & Kim, 1995) **Figure 2.12** **Condition for N* in JIT** $N^*(N^* - 1) \leq \eta/\theta \leq N^*(N^* + 1),$ where $\eta = (A_m M + S)\{h_p(2D/P - 1) + h_r\}$ and $\theta = A_r(D/P)\{h_m/M + h_p(1 - D/P)\}.$ Source: (Banerjee & Kim, 1995) Where A_m = raw material ordering cost, A_r = Supplier's order processing and shipment cost, D = demand rate, h_m = raw material holding cost, h_p = finished goods inventory holding cost, h_r = inventory holding cost for the buyer, M = raw material lot size factor Q_m = NQ/M, N = production lot size factor Q_p= NQ, Q delivery lot size, S = production set-up cost.	
6	A one-vendor multi-buyer integrated inventory model.	Research Paper *European Journal of Operational Research*	Lu Lu	1995	(Lu, 1995) minimize the vendor's total annual cost for single vendor and single buyer with subject to maximum cost that the buyer is ready to pay where the vendor has advantage over the buyer in purchase negotiation and know the buyer's annual demand and order frequency in advance. (Lu, 1995) also had given heuristic approach to minimize the vendor's total annual cost for single vendor and multiple buyers. (Goyal, 1988) model had an assumption that the vendor will only supply items to purchaser after completion of entire production lot. (Lu, 1995) had relaxed this assumption in this research work.	Tried to minimize only the vendor's total annual cost in the integrated model.

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
7	A one-vendor multi-buyer integrated inventory model: A comment.	Research Paper *European Journal of Operational Research*	Goyal, S. K.	1995	(Goyal, 1995) extends work done by (Goyal, 1988) and (Lu, 1995) and given an approach which is capable of giving better relevant total costs of the single vendor-single purchaser production-inventory systems. (Goyal, 1995) had taken ratio of (i+1)th shipment to ith shipment equal to n. Economic Order Quantity, EOQ for k number of lots per production is given in the figure 2.13 and minimum joint total annual cost is given in figure 2.14. **Figure 2.13** **The Economic Order Quantity for k lots/production** $q(k)=\sqrt{\dfrac{2D(S+kA)(n^2-1)}{r(n^{2k}-1)\left(C_Q+\dfrac{C_v}{n}\right)}}$ Source: (Goyal, 1995) **Figure 2.14** **The Minimum Joint Total Annual Cost** $\mathrm{JTRC}(q(k))=\sqrt{\dfrac{2rD(C_Q+C_v/n)(n-1)(n^k+1)(S+kA)}{(n+1)(n^k-1)}}$ Source: (Goyal, 1995) Where JTRC = Joint annual Total Relevant Cost, r = vendor's annual rate of production, P = vendor annual rate of production, S = vendor's setup cost per setup, C_v = vendor's unit manufacturing cost, Q = production lot quantity per production, k = number of shipments per production, D = annual demand rate, P = vendor annual rate of production, n = P / D, C_Q = unit purchase price paid by purchaser, q_i = size of ith shipment.	Extended by (Goyal, 1988) and (Lu, 1995) research and derived better relevant total costs of the single vendor-single purchaser production-inventory systems

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
8	On an inventory model with deteriorating items decreasing time-varying and shortages.	Research Paper *European Journal of Operational Research*	Benkherouf, L.	1995	(Benkherouf, 1995) had given optimal replenishment policy for items that are continuously deteriorating over time at a constant rate and demand rates are deceasing over known time period with shortage in inventory is allowed. Perishable items like food stuff, medicines, volatile liquids, blood banks, etc. are considered in this research work.	(Benkherouf, 1995) discussed inventory policies for perishable items.
9	The single-vendor single-buyer integrated production-inventory model with a generalised policy.	Research Paper *European Journal of Operational Research*	Hill, R. M.	1997	As per (Hill, 1997) none of policies given by (Lu, 1995) and (Goyal, 1995) was have optimal solution. As per him optimal solution could be obtained when successive shipment quantities within a production batch should increased by a fixed factor. The first shipment quantity q* is given in figure 2.15 and mean total cost for q* is given in figure 2.16. **Figure 2.15** **The first shipment quantity** $q^* = \left(\frac{2(A_1 + nA_2) D\lambda(\lambda^2 - 1)}{(h_1 + \lambda h_2)(\lambda^{2n} - 1)} \right)^{1/2}$ Source: (Hill, 1997) **Figure 2.16** **The mean total cost incurred by the system** $C(q^*) = 2\left(\frac{(A_1 + nA_2)(h_1 + \lambda h_2) D}{2} \cdot \frac{(\lambda - 1)(\lambda^n + 1)}{\lambda(\lambda + 1)(\lambda^n - 1)} \right)^{1/2}$ Source: (Hill, 1997) Where A_1 = the fixed production setup cost, A_2 = the fixed order/shipment cost, h_1 = the stockholding cost for the vendor, h_2 = the stockholding cost for the buyer, D = the demand rate, P = the production rate for the vendor, n = the number of	(Hill, 1997) discussed (Lu, 1995) and (Goyal, 1995) models and pointed out these models are not giving optimal solutions.

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					shipments per production run, q = the size of first shipment, λ = the proportional increase in the size of successive shipments, C = the mean cost incurred by the system per unit time, P > D and $h_2 > h1$.	
10	Optimal strategy for the integrated vendor-buyer inventory model	Research Paper *European Journal of Operational Research*	Vishwanathan, S	1998	(Vishwanathan, 1998) discussed two replenishment strategies for integrated vendor-buyer inventory model. The first strategy replenished the buyer's inventory with equal quantity items each time. The second strategy replenished the vendor's inventory with available inventory of an item so that after receiving items the buyer's inventory reached to maximum for the item received. (Vishwanathan, 1998) observed, when there is high ratio value of holding cost of the buyer to holding cost of the vendor, the first strategy of equal item replenishment is more attractive. Higher production rate with respect to demand rate gave less overall cost	As per (Vishwanathan, 1998) equal quantity replenishment is better when the vendor's holding cost is lower than the buyer's. This concept was used by latter research works where equal quantity replenishment strategy had been used
11	An optimal policy for a single-vendor single-buyer integrated production-inventory system with capacity constraint of the transport equipment.	Research Paper *International Journal of Production Economics*	Hoque, M. A. & Goyal, S. K	2000	(Hoque & Goyal, 2000) had developed optimal policy for a single-vendor, single-buyer integrated production system with equal and unequal size batch shipment between stages and limited capacity to transport items.	(Hoque & Goyal, 2000) had examine equal and unequal size shipment lots
12	On optimal two-stage lot sizing and inventory batching policies.	Research Paper *International Journal of*	Hill, R. M.	2000	(Hill, 2000) had discussed coordination between two successive stages of multi stage production system. He had	(Hill, 2000) examined coordination between

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		Production Economics			classified problem as follows **Production rate:** greater than or less than between stages **Production batch size**: greater than or less than between stages. **Items transfer type:** continuous or in batches He had also observed that equal size batches had given better result.	multi stage production system
13	Determination of economic production-shipment policy for a single-vendor-single-buyer system	Research Paper *European Journal of Operational Research*	Goyal, S. K. & Nebebe, F.	2000	According to (Goyal & Nebebe, 2000) had developed a model for single vendor single buyer. The suggested that the first shipment size should be smaller than rest shipment size and equal to (Rate of product/Rate of demand). It would ensure quick delivery after receiving an order and rest (n-1) shipments would be of equal size. They tried to provide simple alternative policy to determine optimal batch quantity for the vendor, economical number of shipments sent from the vendor to the buyer and economical size of shipments. The annual cost for the vendor-buyer had been given as **Figure 2.17** **The total annual cost of the vendor-buyer** $C(n,q) = \frac{(A_1 + nA_2)D}{q(1+(n-1)x)} + \frac{q}{2}\left[h_1 \frac{(2D + (P-D)(1+(n-1)x))}{P} + \frac{(h_2 - h_1)(1+x^2(n-1))}{(1+x(n-1))}\right].$ Source: (Goyal & Nebebe, 2000) Where A_1 = production Set-up cost, A_2 = shipment cost, h_1 = the vendor's holding cost, h_2 = buyer's holding cost, D =	(Goyal&Nebebe, 2000) worked on lead time and tried to reduce by reducing size of first shipment lot.

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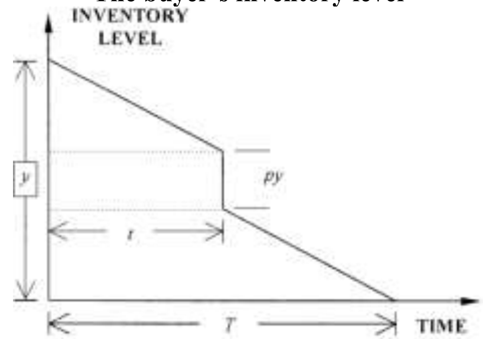
					annual demand rate, P = production rate, x = ratio of production rate to demand rate (i.e., P/D), q = size of first shipment. N = number of shipments, C = total annual cost	
14	Economic production quantity model for items with imperfect quality.	Research Paper *International Journal of Production Economics*	Salameh, M. K. & Jaber, M. Y.	2000	(Salameh & Jaber, 2000) extended the traditional EPQ/EOQ model by taking consideration of imperfect quality items. They considered that 100% screening of items and poor quality would be sold at end of screening process. **Figure 2.18** **The buyer's inventory level**  Source: (Salameh & Jaber, 2000) The Economical Order Quantity suggested by (Salameh & Jaber, 2000) is given in figure 2.19. **Figure 2.19** **The Economical Order Quantity** $y^{*} = \sqrt{\frac{2KD\,E[1/(1-p)]}{h[1-E[p]-D(1-E[1/(1-p)])/x]}}$ Source: (Salameh & Jaber, 2000) Where y = order size, K = ordering cost, p = percentage of defective items, x = screening rate, D = demand rate per year, h = holding cost	(Salameh & Jaber, 2000) did not consider the fact there may be also some error in screening process. Backorder in inventory was also not considered.

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15	Observation on: " Economic production quantity model for items with imperfect quality".	Research Paper *International Journal of Production Economics*	Cárdenas-Barrón L. E.	2000	(Cárdenas-Barrón, 2000) found an error in (Salameh & Jaber, 2000) EOQ formula (given in figure 2.19) and gave the corrected formula as given in figure 2.20. **Figure 2.20** **The Economical Order Quantity** $y^* = \sqrt{\dfrac{2KDE[1/(1-p)]}{h[1-E[p]-(2)(D/x)(1-E[1/(1-p)])]}}$ Source: (Cárdenas-Barrón, 2000) Where y = order size, K = ordering cost, p = percentage of defective items, x = screening rate, D = demand rate per year, h = holding cost	
16	Recent trends in modeling of deteriorating inventory	Research Paper *European Journal of Operation Research*	Goyal, S. K. & Giri, B. C.	2001	(Goyal & Giri, 2001) had classified inventory items into following three categories 1. Obsolescence 2. Deterioration 3. No Obsolescence/Deterioration Obsolescence items are those items which loosed their values due to change in technology or introduction of new product. For example spare parts of an aircraft which has been replaced by new advance aircraft. These spare parts loosed its value. Deterioration items are those items that have very short life and after that they loosed their value. These items are also referred as perishable items. For example foodstuff, green vegetables, human blood, medicine with expiry date are fall into deterioration items categories. (Goyal & Giri, 2001) had discussed inventory models dealing with deterioration items.	

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17	Quality improvement and setup reduction in the joint economic lot size model.	Research Paper *European Journal of Operation Research*	Affisco, J. F., Paknejad, M. J. & Nasri, F.	2002	(Affisco et al., 2002) had discussed co-maker concept in which the supplier and purchaser are value chain partner in a manufacturing process. They discussed following three different cases 1 The basic model as given by (Banerjee, 1986) 2 Quality Improvement 3 Simultaneous quality improvement and setup cost reduction (Affisco et al., 2002) extended the basic model of (Banerjee, 1986) and suggested that the purchaser could go for 100% inspection if the inspection cost is less than the cost of selling defective items. (Affisco et al., 2002) discussed quality improvement of manufacturing process by some investment with an objective to minimize joint total relevant cost (JTRC) and get joint economic lot size (JELS). (Affisco et al., 2002) also discussed simultaneous quality improvement and setup cost reduction. The setup cost reduction allowed smaller JELS. (Affisco et al., 2002) suggested that there should be a continuous quality improvement program and setup cost reduction could be taken as complementary program in manufacturing process	Discussed concept of co-maker in production system and its impact on inventory management
18	An integrated vendor-buyer cooperative	Research Paper *Production*	Huang, C. K.	2002	(Huang, 2002), tried to develop a model to determine an optimal integrated vendor–buyer integrated policy for just-in-	

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	inventory model for items with imperfect quality.	*Planning & Control: The Management of Operations*			time (JIT) environment with an aim to minimize the total annual cost incurred by the vendor and the buyer. The model also taken account of imperfect quality items.	
19	Note on: economic production quantity model for items with imperfect quality–a practical approach.	Research Paper *International Journal of Production Economics*	Goyal, S. K. & Cárdenas-Barrón L. E.	2002	(Goyal & Cárdenas-Barrón, 2002) extended (Salameh & Jaber, 2000) model and developed a simple approach for determining the economic production quantity for an item with imperfect quality through their technical note. The simplified formula is given in figure 2.21. It could be compared with formula given by (Salameh & Jaber, 2000) and latter modified by (Cárdenas-Barrón, 2000) in the figure 2.20 given above **Figure 2.21** **The Economical Order Quantity** $y = y^{**} = \sqrt{\frac{2KDE[1/(1-p)]}{h}}$ Source: (Goyal & Cárdenas-Barrón, 2002).	Extended (Salameh & Jaber, 2000) by Adopting Simple approach for EOQ
20	The economic production lot-sizing problem with imperfect production processes and imperfect maintenance.	Research Paper *International Journal of Production Economics*	Ben-Daya, M.	2002	(Ben-Daya, 2002) developed an integrated model to determine Economics Production Quantity (EPQ) and Preventive Maintenance (PM) level for imperfect production process having a deterioration distribution and increasing hazard rate. He found that performing preventive maintenance (PM) results into reduction quality related cost. As per (Ben-Daya, 2002) when preventive maintenance (PM) cost become higher than reduction quality related cost, further preventive maintenance (PM) is not justified.	Author had given more focus on preventive maintenance (PM) of manufacturing equipments rather than inventory management

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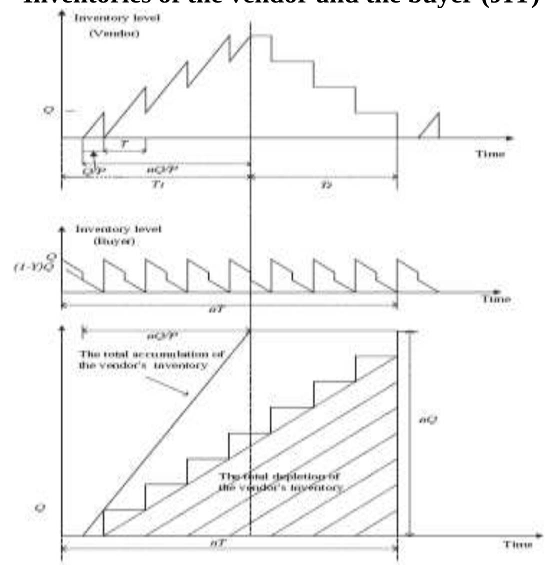
21	An optimal policy for a single-vendor single-buyer integrated production-inventory problem with process unreliability consideration.	Research Paper *International Journal of production economics*	Huang, C. K.	2004	(Huang, 2004) tried to get optimal policy for a single-vendor single-buyer integrated production with process unreliability for Just-in-time (JIT). According to (Huang, 2004) in JIT the buyer had a problem to know how much quantity could be ordered and the vendor had problems to know economic production batch quantity and number of shipments per order. The inventories of the vendor and the buyer model of is given in figure 2.22 **Figure 2.22** **Inventories of the vendor and the buyer (JIT)**  Source: (Huang, 2004) The condition to find optimal number of shipments per lot as per (Huang, 2004) is given in figure 2.23	Inventory Management for JIT.

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					Figure 2.23 **The optimal number of shipments n* in (JIT)** $(n^*-1)n^* \leqslant \frac{(S_V+S_B)\{2DM((h_B/x)+(h_V/P))-h_V-2Dh_B/x+(1-E[Y])h_B\}}{(1-DM/P)Fh_V} \leqslant n^*(n^*+1)$ Source: (Huang, 2004) Using n* as given in figure 2.23, (Huang, 2004) given a formula as shown in the figure 2.24 to calculate the size of optimal shipment Q* from the vendor to the buyer. **Figure 2.24** **The optimal shipment quantity Q*** $Q^*=\sqrt{\frac{2D[(S_V+S_B)/n+F]M}{[2Dh_B/x-(n-2)Dh_V/P]M+(n-1)h_V-2Dh_B/x+h_B(1-E(Y))}}$ Source: (Huang, 2004)	
22	Economic ordering quantity models for items with imperfect quality.	Research Paper *International Journal of Production Economics,*	Papachristos, S. & Konstantaras, I..	2006	(Papachristos & Konstantaras, 2006) had pointed out that conditions for non-shortage of items, as mentioned in (Salamesh & Jaber, 2000) and (Chan et al., 2003) did not really prevent occurrence of shortage in the inventory. (Papachristos & Konstantaras, 2006) extended (Salamesh & Jaber, 2000) model with modified condition and	Pointed there had shortage of items in (Salamesh & Jaber, 2000) and (Chan et al., 2003)
23	Fuzzy economic production quantity model for items with imperfect quality.	Research Paper *International Journal of Innovative Computing, Information and Control*	Chen, S. H., Wang, C.C., & Chang S. M.	2007	(Chen et al., 2007) gave a Fuzzy Economic Production Quantity (FEPQ) model with imperfect products where defective items could be sold at a discount price. In the model cots and quantities represented in fuzzy numbers. They used Graded Mean Integration Representation method to defuzzing and Kuhn-Tucker conditions to find optimal economic production quantity.	Fuzzy model for imperfect products

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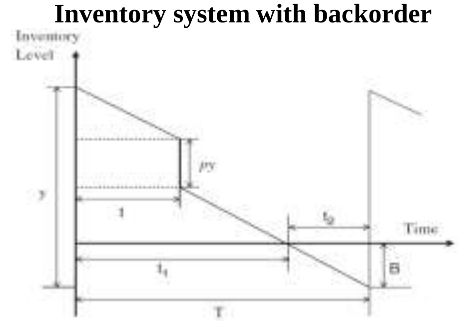
					(Chen et al., 2007) had provided following equation as mentioned in figure 2.25 to calculate Optimal Production Quantity **Figure 2.25** **The optimal production quantity Q*** $Q^* = \sqrt{\dfrac{2(d_1k_1 + 2d_2k_2 + 2d_3k_3 + d_4k_4)}{2p(d_1h_1 + 2d_2h_2 + 2d_3h_3 + d_4h_4) + (1-p)^2(h_1 + 2h_2 + 2h_3 + h_4)}}$ Source: (Chen et al., 2007) Where (h1,h2,h3,h4) = fuzzy holding cost, (k1,k2,k3,k4) = fuzzy setup cost, (d1,d2,d3,d4) = fuzzy demand, p = the percentage of defective items in a production lot,	
24	Optimal inventory model for items with imperfect quality and shortage backordering	Research Paper *Omega*	H. M. Wee, Jonas Yu and M. C. Chen	2007	This research work generalized production lot size model with backordering. It extended the approach of (Salameh & Jaber, 2000) by considering permissible shortage backordering and the effect of varying backordering cost values. It introduced the concept of backorder due to imperfect quality of production. **Figure 2.26** **Inventory system with backorder**  Source: (Wee. et al., 2007)	The backorder level reached to zero on arrival of fresh lot of items. In this research work inspection process is done at the buyer's site after receiving a new lot of items

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					Figure 2.27 **Optimal Order Size y* and Optimal Back Order Size B*** $y^* = \sqrt{\frac{(2DK + B^2h + B^2b)E[1/(1-p)]}{h(1 - E[p] - 2(D/x)(1 - E[1/(1-p)]))}},$ $B^* = \frac{h}{(h+b)(1/\beta - \alpha)\ln((1-\alpha)/(1-\beta))}.$ Source: (Wee. et al., 2007) Where y = order size, D = demand rate, x = screening rate, K = ordering cost, B = maximum backorder quantity allowed, h = inventory holding cost, p = defective percentage, α = minimum value for p, β = maximum value for p.	
25	Economic order quantity for items with imperfect quality: revisited.	Research Paper *International Journal of Production Economics*	Maddah, B. & Jaber, M. Y.	2008	(Maddah & Jaber, 2008) considered imperfect quality items and screening process as random function and analyzed (Salmesh & Jaber, 2000) model using renewal theory. They found that effect of screening speed and variation in supply process due to random imperfect items, the order quantity calculated by them was larger than (Salmesh & Jaber, 2000) model and the same profit was also found lesser. The optimal order quantity is given in figure 2.26. The optimal order quantity of (Salmesh & Jaber, 2000) has been given in figure 2.19 which was corrected latter by (Cárdenas-Barrón, 2000) figure 2.20. **Figure 2.28** **The Economical Order Quantity** $y^* = \sqrt{\frac{2KD}{h[E[(1-P)^2] + 2E[P]D/x]}}$ Source: (Maddah & Jaber, 2008)	Used renewal theory

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					Where K = Ordering cost, D = Demand Rate, h = inventory holding cost, x = rate of inspection, y = order size, P = fraction of defective items in a lot.	
26	Exact closed-form solutions for ‘‘optimal inventory model for items with imperfect quality and shortage backordering’’	Research Paper *Omega*	Chang, H. C. & Ho, C. H.	2010	(Chang & Ho, 2010) revisit (Wee et al., 2007) and apply the well-known renewal- reward theorem to obtain a new expected net profit per unit time (gives better result). They also provided an approach to solve the same problem algebraically from another direction.	Discussed imperfect product with shortage backorder
27	An economic order quantity (EOQ) for items with imperfect quality and inspection errors	Research Paper *International Journal of Production Economics*	Khan, M., Jaber, M. Y. & Bonney, M.	2011	(Khan et al., 2011) extend the work of (Salameh & Jaber, 2000) and introduce the concept that inspection of items for defects can also have errors. A defective item can be classified as non-defective and non-defective item can also classify as defective. There are two types of inspection errors Type I Error: An inspector may classify a non-defective item as defective Type II Error: An inspector may classify a defective item as non-defective B2 defective items classified as non-defective would be sold in market and latter replaced and stored in inventory.	Discussed imperfect product with shortage backorder and inspection error

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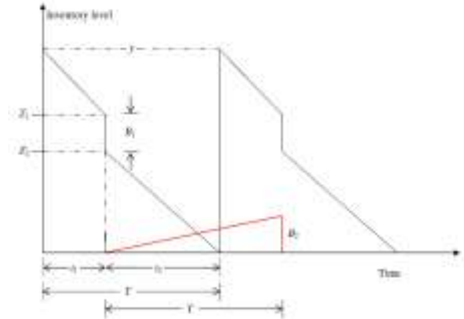
					Figure 2.29 **Inventory Level over time**  Source: (Khan et el., 2011) (Khan et el., 2011 derived the formula for calculating the expected annual profit as given in the figure 2.30 and EOQ as given in the figure 2.31. **Figure 2.30** **Expected annual profit** $E[TPU(y)] = sD + \frac{sDE[p]E[m_2]}{(1-E[p])(1-E[m_1])} + \frac{vDE[m_1]}{(1-E[m_1])} + \frac{vDE[p]}{(1-E[p])(1-E[m_1])} - \frac{D\left[\frac{K}{y} + c + d + c_r(1-E[p])E[m_1] + c_a E[p]E[m_2] + \frac{h}{2}\left\{\left(\frac{2}{x} - \frac{D}{x^2} + \frac{E[A^2]}{D}\right)y\right\}\right]}{(1-E[p])(1-E[m_1])} - h\frac{yE[p]E[m_2]}{2}$ Source: (Khan et el., 2011) **Figure 2.31** **Economic Order Quantity (EOQ)** $y^* = \sqrt{\frac{2KD}{hE[p]E[m_2](1-E[p])(1-E[m_1]) + hD\left(\frac{2}{x} - \frac{D}{x^2} + \frac{E[A^2]}{D}\right)}}$ Source: (Khan et el., 2011)	

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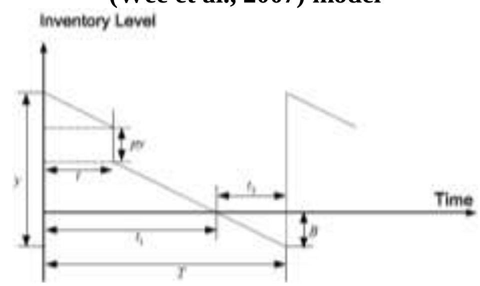
28	Disaggregation and consolidation of imperfect quality shipments in an extended EPQ model	Research Paper *International Journal of Production Economics*	Yassine, A., Maddah, B. & Salameh, M.	2012	(Yassine et al., 2012) discussed a tradeoff between disaggregation of imperfect quality items shipment and shipped multiple imperfect items during a production cycle vs. consolidation of imperfect quality items shipment over multiple production cycle where imperfect items for multiple production cycle. (Yassine et al., 2012) showed that disaggregation of imperfect quality items shipment reduced overall inventory management cost.	There is no change in assumptions as made by (Salameh & Jaber, 2000)
29	A note on "Optimal inventory model for items with imperfect quality and shortage backordering"	Research Paper *International Journal of Industrial Engineering Computations*	Hsu, J. & Hsu, L. (2012b)	2012	In (Wee et al., 2007) backorder get clear as soon as new batch of items arrived. It did not consider that inspection of items need some time. (Hsu & Hsu, 2012b) had corrected this problem and proposed a model **Figure 2.32** **Inventory system with complete backordering (Wee et al., 2007) model**  Source: (Hsu & Hsu, 2012b)	Discussed imperfect product with shortage backorder

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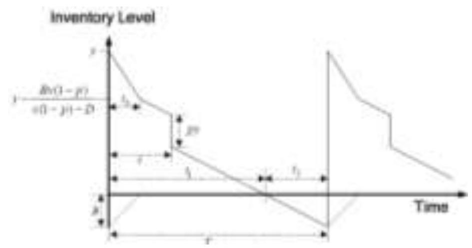
					Figure 2.33 **Behavior of the inventory level over time for the model corrected by (Hsu & Hsu, 2012b)**  Source: (Hsu & Hsu, 2012b) They found the economic order quantity (EOQ) and optimal backorder quantity allowed as shown in the figure 2.34 and expected total profit per unit time as given in figure 2.35 **Figure 2.34** **Economic Order Quantity (EOQ)** $y^* = \sqrt{\dfrac{2KD}{h\left\{E[(1-p)^2]-R^2A_1+2E[p]\dfrac{D}{x}\right\}-bR^2\left(1+A_3\dfrac{D}{x}\right)}}$ $B^* = y^* R$ Where $R = \dfrac{h(1-E[p]-A_1D/x+A_2)}{2(hA_1+b+bA_3D/x)}$ Source: (Hsu & Hsu, 2012b)	

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					Figure 2.35 **Expected Total Profit per unit time (ETPU)** $ETPU(B,y) = sD + vD\frac{E[p]}{(1-E[p])} - \frac{KD}{(1-E[p])y} - \frac{cD}{(1-E[p])} - \frac{dD}{(1-E[p])} - \frac{1}{2}h\frac{DB}{x(1-E[p])}E\left[\frac{(1-p)}{(1-p-\frac{D}{x})}\right]$ $-\frac{1}{2}h\left(\frac{yE[(1-p)^2]}{(1-E[p])} - \frac{B}{(1-E[p])}E\left[\frac{(1-p)^2}{(1-p-\frac{D}{x})}\right] - B + \frac{B^2}{y(1-E[p])}E\left[\frac{(1-p)}{(1-p-\frac{D}{x})}\right]\right) - h\frac{E[p]yD}{x(1-E[p])}$ $-\frac{1}{2}bB^2\left(\frac{1}{y(1-E[p])} + \frac{D}{xy(1-E[p])}E\left[\frac{1}{(1-p-D/x)}\right]\right)$ Source: (Hsu & Hsu, 2012b)	
30	Lot sizing in case of defective items with investments to increase the speed of quality control	Research Paper *Omega*	Hauck, Z., Vörös, J. (2015)	2015	(Hauck, 2015) stated that increasing the speed of inspection process enabled the system to respond fast and save money. (Hauck, 2015) had developed two models. The first model always remain in the same state while in second model the percentage of defective items was different in consecutive lots and the same time speed of inspection of items was also different. (Hauck, 2015) had stated that increasing speed of inspection process is controversial. Increasing the speed reduce total inventory management cost but when percentage of defective items was high and there was backlog of items then it increased total inventory management cost	(Hauck, 2015) stared that impact of increasing inspection process speed need more research work and that would enable to get optimal inspection process speed.
31	Optimal Buyer's Replenishment Policy in the Integrated Inventory Model for Imperfect Items.	Research Paper *Mathematical Problems in Engineering*	Yueli, L., Jiangtao M. & Yucheng W.	2016	(Yueli & Yucheng, 2016) extended (Maddah & Jaber, 2008) model by making assumption that ordering cycle would be based on demand rate, number of items in a lot and mathematical exception for rate of defects in a lot. By adding these assumptions they had discussed possibilities of shortage of items due to random defective items in lots. They had taken	(Yueli & Yucheng, 2016) pointed the possibility of shortage of items at buyer end as defective percentage is a random variable.

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					two cases. For the first case, extra items were added in lots to avoid shortage for second case they let the shortage happen. For these conditions they tried to find optimal ordering cycle.	
32	Integrated supply chain inventory model with quality improvement involving controllable lead time and backorder price	Research Paper *International Journal of Industrial Engineering Computations*	Jindal, P. & Solanki, A.	2016	(Jindal & Solanki, 2016) had discussed continuous review inventory management and considered order quantity, reorder point, lead time, process quality and backorder price discount and number of shipments as decision variables and tried to minimize total related cost of inventory. (Jindal & Solanki, 2016) had made assumption that buyer was motivating consumers to wait for possible backorder by giving price discount. They also assumed that items received from the vendor contain defective items. They tried to get optimal values for decision variable by using iterative method to minimize total expected cost.	Discussed continuous review of inventory with order quantity, reorder point, lead time, process quality and backorder price discount and number of shipments as decision variables
33	Inventory Modeling for Imperfect Production Process with Inspection Errors, Sales Return, and Imperfect Rework Process.	Research Paper *International Journal of Mathematical, Engineering and Management Sciences*	Khanna, A., Kishore, A. & Jaggi, C. K.	2017	(Khanna et al., 2017) tried to minimize losses occurred due to production of defective items and proposed reworking on defective items to remove defects. They tried to consider human error is a reality of life and consider that the rework process was also imperfect. To improve consumer satisfaction they assumed 100% full price return to consumer on sales return due to manufacturing defects. They tried to maximize the expected total profit per unit time. The pertinence of the model can be found in most manufacturing industries like textile, electronics, furniture, footwear, crockery etc.	Proposed reworking of defective items for imperfect production and inspection error

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
34	An Integrated Imperfect Production–Inventory Model with Optimal Vendor Investment and Backorder Price Discount.	Research Paper *Information Technology and Applied Mathematics*	Mukherjee, A., Dey, O. & Giri, B.C.	2019	(Mukherjee et al., 2019) developed an imperfect inventory for integrated single-vendor, single-buyer where the vendor make investment to improve quality of items during production and same time the buyer offered price discount to consumers for backorder as incentive so that consumers could wait for some time to get their item. Their inventory management had followed continuous review by the buyer to place order of items in place of periodic review policy adopted by most of inventory management models. The lot size for an order depends upon lead time, backorders and lost sales. (Mukherjee et al., 2019) derived the optimal expected annual total cost of the integrated system using n-shipment policy.	Discussed investment to improve quality and price discount as incentive for shortage of items.
35	Joint replenishment decision considering shortages, partial demand substitution, and defective items	Research Paper *Computer& Industrial Engineering*	Yanru Chen, Lu Yang, Yangsheng Jiang, M.I.M. Wahabe, Jie Yang	2019	(Chen et al. 2019) discussed a joint replenishment problem (JRP) where shortage of items can be partially fulfilled with substitutable items which may occurs insufficient production capacity or possible damages of items in transit. (Chen et al. 2019) considered real-world constraints such as budget, transportation capacity, and shipment requirement constraints. They deceloped three different algorithms for it, two-dimensional genetic algorithm I, two-dimensional genetic algorithm II, and differential evolution.	
36	Economic order quantity model for growing items with	Research Paper *Operations Research*	Makoena Sebatjane, Olufemi	2019	(Sebatjane & Adetunji, 2019) proposed an inventory system where ordered items are growing during inventory replenishment cycle like livestock items. As livestock grow,	

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
	imperfect quality	*Perspectives*	Adetunji		the demand will also grow. They assumed that there are certain fractions of items are of poor quality. Using these assumptions they tried to maximize total profit.	

2.5 The Research Gap

Going through the review of literature, it has been found that inventory management models had been improved by various research works. These models covered diversified aspects of inventory management. Imperfect quality production as well as imperfect inspection process has added on to the research area of inventory management models. These areas of research work have a lot of attention from researchers. (Salameh & Jaber, 2000), first considered imperfect inspection process, and proposed a model, where the inspection process done by the buyer, after receiving a fresh items' lot from the vendor. (Wee et al., 2007) extended (Salameh & Jaber, 2000) by considering backorder quantities in the buyer's inventory of the supply chain management. (Khan et al., 2011) had added imperfect inspection process to inventory management. Research works that followed (Salameh & Jaber, 2000) continued to consider the same assumptions. This has found a gap in the research works. This thesis is an attempt to change the old assumptions of performing inspection presses by the buyer, by a new assumption that the inspection process is done, by the vendor along with the production of items. The research work compare integrated inventory management with the buyer's independent decision model with new assumption and done sensitivity analysis of cost parameters on total expected cost of inventory management. This will help manufacring industries to take decision related to their inventory management and develop strategies for it.

2.6 Summary of Literature Review

Table 2.2: Summary of literature review

Srl No	Broad Topic	Literature Surveyed
1	Integrated Single Vendor – Single Buyer with make to order Inventory Management	2
2	Integrated Single Vendor – Single Buyer with backorder Inventory Management	3
3	Integrated Single Vendor – Single Buyer Inventory Management	29
4	Integrated Single Vendor – Multiple Buyer with backorder Inventory Management	6
5	Integrated Single Vendor – Multiple Buyer Inventory Management	14
6	Integrated Multiple Vendor – Single Buyer Inventory Management	3
7	Integrated Multiple Supplier – Single Vendor Inventory Management	3
8	Multi Stage Inventory Management	5
9	Multi Product Inventory Management	3
10	Inventory Management with variable setup cost	2
11	Inventory Management with transport capacity constraints	2
12	Inventory Management with quantity discount	5
13	Inventory Management with limited inventory storage capacity	1
14	Inventory Management with limited capital	1
15	Inventory Management with lead time reduction	4
16	Inventory Management with Fright discount	3
17	Inventory Management with dynamic demand	11
18	Inventory Management with delay in payment	6
19	Inventory Management using RFID	2
20	Inventory Management of Just in Time (JIT)	3
21	Inventory Management for perishable (short life span) items	3
22	Imperfect production quality with production system inspection	13
23	Imperfect production quality Inventory Management with backorder	8
24	Imperfect production quality Inventory Management	15
25	Imperfect production quality and rework Inventory Management	4
26	Imperfect production quality and Imperfect inspection Inventory Management	9
27	Fuzzy Inventory models	11
	Total	171

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

Mathematical modeling is used in this research. Mathematical modeling converts real-world phenomena in mathematical equations and mathematical tools are used to solve the real-world problem. Optimization of the expected total cost is done. To find optimal values from a mathematical function curve, nature of the curve is checked. The strictly convex curve has a minimum value and strictly concave curve has a maximum value. The theory of Exception and Renewal and Reward theorem are also used for finding optimal values.

3.2 Introduction to Mathematical Modeling

The real world situation usually has many facets. Mathematical modeling translates these facets of a real world situation in corresponding with mathematical terms. Theories and algorithms of mathematics are used to get insights into the situation, try to solve problems and translate back the model back into a real world situation, (Marion, 2008).

Mathematical modeling is used for this research work and hence fourth developed mathematical models for management of inventory in supply chain management.

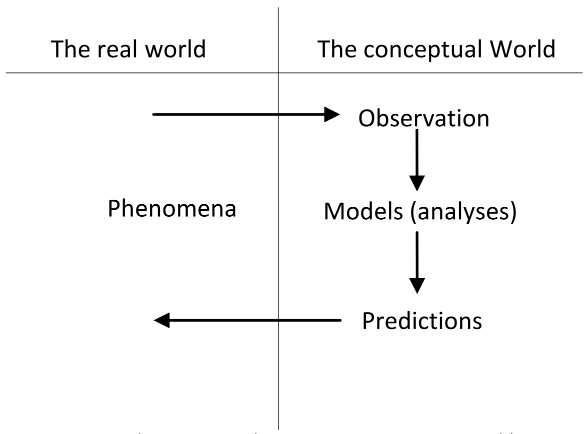
Mathematical model has following advantages:

- Mathematics is a well-defined language. Which helps to identify assumptions and formulate them.
- Mathematics has well-defined rules and theorem for manipulation of mathematical models.
- The results of mathematical manipulation have proved over hundreds of years.

- Mathematical calculations can be performed very efficiently by using computers.

There are various types of mathematical models. These are Deterministic/Stochastic and Mechanistic/Empirical. Deterministic models deal with exact values where stochastic models deal with probable values which have a probability distribution. Mechanistic models use a large amount of theoretical information and describe what will happen. Empirical models take account of system changes quantitatively and try to give solutions with different conditions

Figure 3.1 Depiction of conceptual models



(Source: (Dym & Ivey, 1980))

Following chart shows a comparison between different types of models.

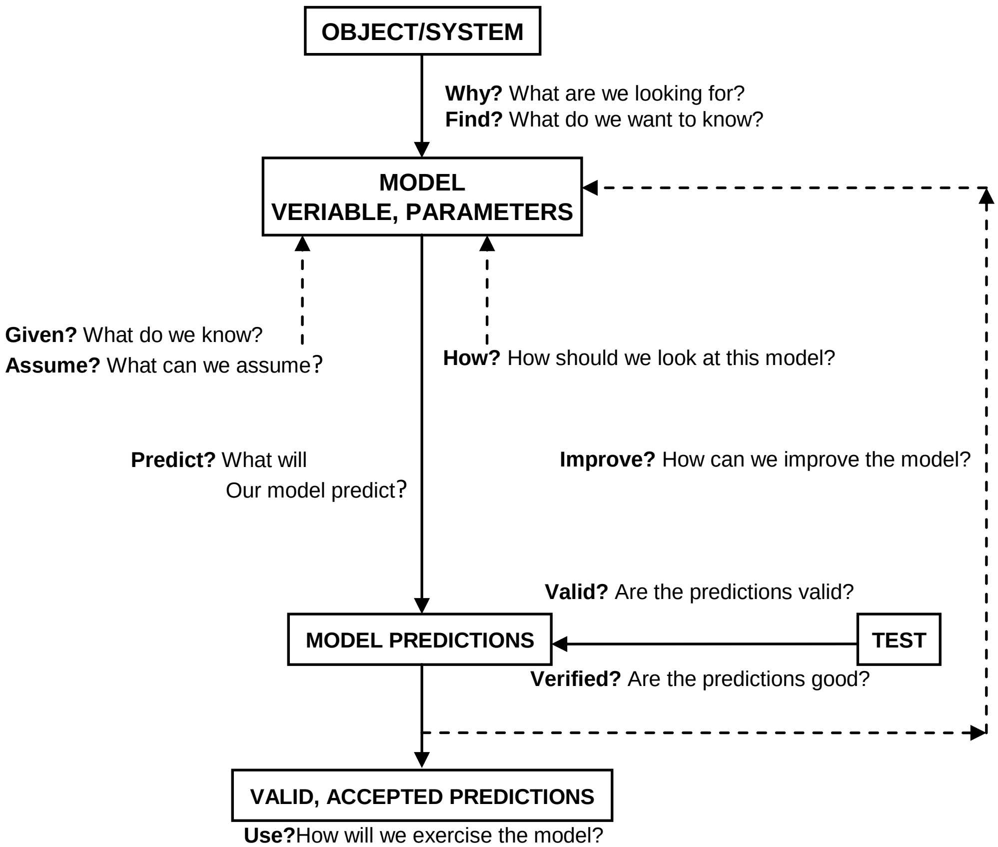
Table 3.1 Types of Mathematical Models

	Empirical	**Mechanistic**
Deterministic	Analysis growth of a child with respect to feed intake	Motion of the Earth around the Sun
Stochastic	Analysis of different variety of crop yield from different sites and years	Food habit of a small village

(Source: (Marion, 2008))

Figure 3.2 explains the philosophical approach of model building and it also gives a list of questions and answers which should be asked in this approach. When these questions will be asked and what are their objectives are given in the figure 3.2.

Figure 3.2
Philosophical approach of model building
The figure is inspired by (Carson & Cobelli, 2001)



(Source: (Marion, 2008))

3.3 Steps of mathematical modeling

Following are the steps of modeling as explained by (Marion, 2008):

3.3.1 Building of model

3.3.1.1 Making assumptions

The model building process starts with making assumptions and drawing flow diagram. Real world workings are translated into assumptions. These assumptions make the structure of the model. Assumptions must be precise and non-ambiguous. Later, these are translated into mathematical equations. When the system is very complex visual flow diagram is used to explain the model visually. The structure gives information about the level of details that have been included in model. It can be empirical, mechanistic, deterministic or stochastic model, (Marion, 2008).

The structure of vendor-buyer Supply Chain Management without backorder, one of the models developed in this thesis, is explained by following figures 3.3 and 3.4

Figure 3.3 Vendor's Inventory

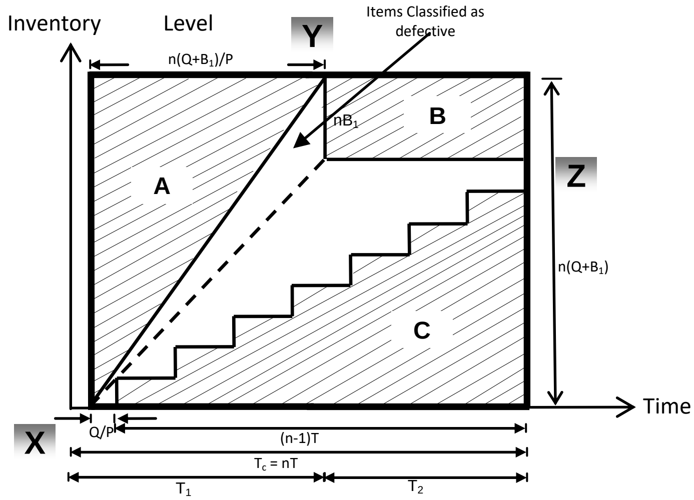
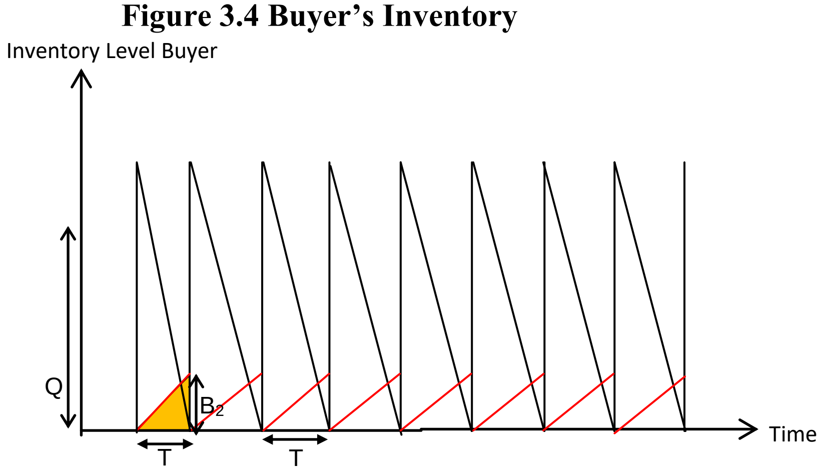


Figure 3.4 Buyer's Inventory



Following assumptions are made for models.

Q_P : the count of items of a lot that are produced per production cycle

Q : the count of items of a lot that delivered from vendor to buyer

n :the number of deliveries to buyer per production cycle, a positive integer $(Q_P = nQ)$

D :the demand rate of items per year

P :the production rate of items (P > D)

x :the Items inspection rate of items x > P

S_v :the Setup cost per production cycle

K :the ordering cost per order for the buyer

C_i :the vendor's inspection cost per unit

C_w :the vendor's cost per unit for producing defective items (warranty cost)

$C_{\alpha\beta}$:the buyer's cost per unit for selling defective items in the market (due to Type II Error)

C_{av} :the vendor's cost of selling defective items in the market (due to Type II Error)

C_r :the vendor's cost for rejecting non-defective items as defective items

h_v :the inventory holding cost per unit item of vendors

h_b :the inventory holding cost per unit item for buyer

F :the transportation cost per delivery of items

p :the probability of production of defective items

e_1 :the probability of type I inspection Error (Classifying Non-Defective item as Defective items)

e_2 :the probability of type II inspection Error (Classifying Defective item as Non-Defective items)

B_1 :the number of items classified as defective after inspection per production lot

B_2 :the number of defective items classified as Non-Defective item per production lot (Type II Inspection Error)

T :the time interval between two successive deliveries to buyers of Q items

T_1	:the time period during which vendor produce items
T_2	:the time period during which vendor supplies items from inventory
T_C	:the production cycle time ($T_C = nT$)
*	:the superscript to represent optimal value

- The inventory model is for non-perishable items which has a long life span.
- It is single-vendor and single-buyer model.
- The rate of production of items "P" is greater than the rate of demand "D" i.e., P > D.
- Production lot is greater than supply lot and used to supply number lots to the buyer to meet demand. Number of lots "n" effects lot size and total cost of inventory. The optimal value of "n" is determined by the model.
- "T" is the time duration between two consecutive supplies to the buyer.
- The production process is of imperfect quality and produces some defective items with probability of "p".
- 100% inspection of items has been conducted. The inspection process is also assumed to be imperfect and there are some errors during the inspection. An inspector may classify non-defective items as defective items (Type I inspection error) with probability e_1 or defective items as non-defective items (Type II inspection error) with probability e_2.

- The inspection rate "x" is greater than the production rate "P" i.e., $x > P$.
- Items classified as defective are disposed at discount rate.
- Defective items B_2, which are classified as non-defective items by an inspector (type II inspection error), sold at market and later returned back by the consumer under warranty, are sent back to the vendor and disposed at discounted rate.
- As there are some items being produced defective, B_1 additional items are produced for each lot items Q making it to $Q + B_1$ items.
- In this research work, first two models (without backorder – Integrated Model and The buyer's independent decision model) do not allow a shortage of items in the inventory and next two models (with backorder – Integrated Model and The buyer's independent decision model) allow a shortage of items with consent from the buyer that he/she will wait for fresh lot items to arrive.

3.3.1.2 Choosing mathematical equations

After the structure of the model is developed, the structure will be converted into mathematical equations. Converted equations will describe the model in mathematical terms.

If the similar structure had been already converted into mathematical equation in previous literatures, then those conversions are taken from the literature review carefully.

A lot of concepts from the field of physics have been converted into mathematical equations and used to solve complex problems. Help from physics also helps to convert the model structure into mathematical equations.

Sometimes structure is new or too complex to convert into equations and there is no help from literature review. In such situation, relevant data is collected for making a structure. Proposed equation is examined it must match with the data collected.

In the figure 3.3 a rectangle is represented by bold lines. As shown in the figure, the height of the rectangle is $n(Q + B_1)$ representing $Q + B_1$ items for n lots and width is Q/P + (n-1)T representing Q/P time taken for the first lot and (n-1)T for rest n-1 lots. The complete rectangle area represents $n(Q + B_1)$ items for Q/P + (n-1)T time.

As shown in the figure, the production process starts from T – Q/P time and the inventory start growing. The growth of inventory is shown by line X to Y. At the same time inspection of the items is also being conducted. During the inspection some items are classified as defective items. The total inventory of items contains both defective items and non-defective items. Thus level of non-defective is less than the total inventory level. This reduction of non-items is shown by dashed line from X to Y in the above figure 3.3. When production ends, nB1 items that classified as defective have been removed from inventory. This is shown a line dipping at Y. The stair like line from X to Z represents supply of Q items at regular intervals. Thus only the plain area of the rectangle represent inventory for the vendor. To get inventory, shaded area A, B and C are deducted from area of the rectangle.

Thus inventory of the vendor is

Area of rectangle = $n(Q+B_1)\left\{\frac{Q}{P}+(n\text{-}1)T\right\}$

Area of A = $\frac{1}{2}n(Q+B_1)\ \frac{n(Q+B_1)}{P}$

Area of B = $\left\{\frac{Q}{P}+(n\text{-}1)T\text{-}\ \frac{n(Q+B_1)}{p}\right\}nB_1$

Area of C = $\frac{n(n\text{-}1)TQ}{2}$

So, inventory of the vendor = bold area – shaded area A, B and C

$$= n(Q+B_1)\left\{\frac{Q}{P}+(n\text{-}1)T\right\}\text{-}\ \frac{1}{2}n(Q+B_1)\ \frac{n(Q+B_1)}{P}\text{-}\ \left\{\frac{Q}{P}+(n\text{-}1)T\text{-}\ \frac{n(Q+B_1)}{p}\right\}nB_1\text{-}\ \frac{n(n\text{-}1)TQ}{2}$$

(3.1)

After multiply, the inventory holding cost per unit item h_v to equation 3.1, the inventory carrying cost for the vendor, has been arrive as given in equation 3.2. "Inventory carrying cost (vendor) =

$$h_v\left[n(Q+B_1)\left\{\frac{Q}{P}+(n-1)T\right\}-\frac{1}{2}n(Q+B_1)\ \frac{n(Q+B_1)}{P}-\left\{\frac{Q}{P}+(n-1)T-\frac{n(Q+B_1)}{p}\right\}nB_1-\frac{n(n-1)TQ}{2}\right]$$

After solving above

$$= \frac{h_v}{2P}\ [\,(2n-n^2)Q^2+n(n\text{-}1)PTQ+n^2{B_1}^2] \tag{3.2}$$

B_1 items are found defective after inspection of $(Q + B_1)$ items with p probability of defective items during production, e_1 type I inspection error and e_2 type II inspection error. The value of B_1 is expressed in mathematical form as

$$B_1 = (Q+B_1)p(1-e_2)+(Q+B_1)(1-p)e_1 \tag{3.3}$$

after solving equation 3.3, value of B_1 could be calculated by formula given in equation 3.4

$$B_1=\frac{p(1-e_2)+(1-p)e_1}{1-\{p(1-e_2)+(1-p)e_1\}}Q \quad (3.4)$$

B_2 items are classified as non-defective items due to type II inspection error. For a lot of Q items Q + B_1 items are produced. (Q + B_1)p items will be produce defective from Q + B_1 items with probability p to produce defective item. (Q + B_1)pe2 defective items will be as non-defective items. Thus

$$B_2=(Q+B_1)pe_2 \quad (3.5)$$

After putting value of B_1 in equation

$$B_2=\frac{pe_2Q}{1-\{p(1-e_2)+(1-p)e_1\}} \quad (3.6)$$

These B_2 items are sold in the market and latter returned back be consumer. These items create a replacement demands addition to the market demand D. As B_2 items are required for T time duration, the demand could be expressed as $\frac{B_2}{D}$. Thus effective demand will be $D' = D + \frac{B_2}{T}$. The cycle time of each delivery T can be calculated by $Time = \frac{Lot\ Size}{Effective\ Demand}$

So, $T = \frac{Q}{D\prime}$ Substituting value of D' $T = \frac{Q}{D+\frac{B_2}{T}}$. Substituting value of B2 from 3.6 and solving

$$T=\frac{(1-p)(1-e_1)Q}{[1-\{p(1-e_2)+(1-p)e_1\}]D} \quad (3.7)$$

Same way

Setup Cost = S_v

Warranty Cost = n(Q + B_1)pC_w $= \frac{npc_wQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{npc_wQ}{A}$

Type I Error = n(Q + B_1) (1 – p) e_1C_r $= \frac{n(1-p)e_1C_rQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{n(1-p)e_1C_rQ}{A}$

Type II Error = n(Q + B_1)pe_2C_{av} $= \frac{npe_2C_{av}Q}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{npe_2C_{av}Q}{A}$

Inspection Cost = $n(Q + B_1)C_i \qquad = \frac{nC_iQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{nC_iQ}{A}$

Thus total cost for the vendor $TC_v(n, Q)$ is

$$TC_v(n,Q) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\Big[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\Big] \tag{3.8}$$

Where $A = 1 - \{p(1-e_2) + (1-p)e_1\}$ (3.9)

The total cost for the vendor depends on dependent variable Q and n.

Following is calculation of holding cost for the buyer. Q items arrived at start of lot time T. Level of inventory came to 0 (zero) at the end of the lot time. So, the average inventory level for fresh items is Q/2. Similarly level of defective items are 0 (zero) at beginning and reached to B_2 at end of the cycle time. Average inventory level of defective items is $B_2/2$. For one production cycle n lots are supplied to the buyer. For one production cycle inventory holding cost HC_b (h_b is holding cost per unit per unit time) for the buyer is

$$HC_b = n * (h_b \frac{Q}{2}T + h_b \frac{B_2}{2}T)$$

After putting values of B2 (from equation 3.6) and T (from equation 3.7) the inventory holding cost HC_b for the buyer is

$$HC_b = \frac{nh_b}{2D} \frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{[1-\{p(1-e_2)+(1-p)e_1\}]^2} Q^2$$

After adding other cost the total cost $TC_b(n,Q)$ for the buyer is

$$TC_b(n,Q) = K + nF + \frac{nc_{\alpha\beta}pe_2\,Q}{1-\{p(1-e_2)+(1-p)e_1\}} + \frac{nh_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{[1-\{p(1-e_2)+(1-p)e_1\}]^2}Q^2 \tag{3.10}$$

The total cost for the buyer depends on dependent variable Q and n.

Equation 3.8 gives total cost of the vendor and equation 3.9 gives total cost of the buyer without back order. Adding both gives integrated total cost of the inventory management. The integrated total cost is

$$TC_C(n,Q) = TC_v(n,Q) + TC_b(n,Q)$$

$$TC_C(n,Q) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\Big[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2) + (1-p)e_1\}^2Q^2}{A^2}\Big] + K + nF + \frac{nc_{\alpha\beta}pe_2\,Q}{A} + \frac{nh_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{A^2}Q^2 \quad (3.11)$$

The buyer can take a decision that he/she is not going to cooperate with the vendor to reduce overall cost, but take own decision and place an order and needs all ordered quantity in a single lot. We refer this situation as "The buyer's independent decision". For this situation by taking n=1, the total cost of the inventory for the buyer will be

$$TC_b(Q) = K + F + \frac{c_{\alpha\beta}pe_2\,Q}{A} + \frac{h_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{A^2}Q^2 \quad (3.12)$$

For the models allowing backorder, backorder cost is also considered.

The vendor's total cost is

$$TC_v(n,Q) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\Big[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2) + (1-p)e_1\}^2Q^2}{A^2}\Big] \quad (3.13)$$

The buyers total cost is

$$TC_b(n,Q,B_3) = K + nF + \frac{nc_{\alpha\beta}pe_2\,Q}{1-\{p(1-e_2)+(1-p)e_1\}} + \frac{nbB_3^2}{2D} + \frac{nh_b}{2D}\Big\{(Q-B_3)^2 + \frac{(1-p)(1-e_1)pe_2Q^2}{[1-\{(1-e_2)+(1-p)e_1\}]^2}\Big\} \quad (3.14)$$

After adding 3.13 and 3.14

$$TC_C(n,Q,B_3) = S_v + K + nF + \frac{nbB_3^2}{2D} + \frac{nh_b(Q-B_3)^2}{2D} + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} +$$

$$\frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{nc_{\alpha\beta}pe_2\,Q}{A} + \frac{h_v}{2P}\Big[\big(2n - n^2\big)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} +$$

$$\frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\Big] + \frac{nh_b(1-p)(1-e_1)pe_2Q^2}{2A^2D} \qquad (3.15)$$

The total cost for the buyer depends on dependent variable Q, n and B_3. Here B_3 is optimal backorder quantity that is allowed.

The buyer can take a decision that he/she is not going to cooperate with the vendor to reduce overall cost, but take own decision and place an order and needs all ordered quantity in a single lot. We refer this situation as “The buyer’s independent decision”. For this situation by taking n=1, the total cost of the inventory for the buyer will be

$$TC_b(Q,B_3) = K + F\ \frac{C_{\alpha\beta}pe_2Q}{A} + \frac{bB_3^2}{2D} + \frac{h_b}{2D}\Big[(Q-B_3)^2 + \frac{(1-p)(1-e_1)pe_2Q^2}{A^2}\Big] \qquad (3.16)$$

3.3.1.3 Solving equations

After conversion of model structure into mathematical equations, these equations are used to solve model objective. A stochastic model outcome is not precise and provides analytical solutions in term of distribution as it allows manipulations on the model with minimum confusion and simulation is used to show the results. Where as a deterministic model outcome is precise and provides numerical solution.

Equation 3.11 gives total inventory cost of integrated inventory management and equation 3.12 gives total inventory cost of the buyer’s independent decision management where backorder has been not allowed. Similarly equation 3.15 gives total inventory cost of integrated inventory management and equation 3.16 gives total inventory cost of the buyer's independent decision management where backorder has been allowed.

After solving and applying theory of expectation the equation 3.11 becomes

$$E[TC_C(n,Q)] = S_v + K + nF + \Big[nc_w E[p] + nC_r(1-E[p])E[e_1] + nC_{av}E[p]E[e_2] + nC_i + nc_{\alpha\beta}E[p]E[e_2]\Big]\frac{Q}{E[A]} +$$

$$\left[\frac{h_v}{2P}\left\{(2n - n^2) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{E[A]D} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right\} + \frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}\right]Q^2 \qquad (3.17)$$

After solving and applying theory of expectation the equation 3.12 becomes

$$E[TC_b(Q)] = K + F + \frac{c_{\alpha\beta}E[p]E[e_2]Q}{[A]} + \frac{h_b}{2D}\frac{[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{E[A^2]}Q^2 \qquad (3.18)$$

After solving and applying theory of expectation the equation 3.15 becomes

$$E[TC_C(n,Q,B_3)] = S_v + K + nF + \frac{n(b+h_b)B_3^2}{2D} + \frac{nc_wQE[p]}{E[A]} + \frac{nC_rQ(1-E[p])E[e_1]}{E[A]} + \frac{nC_{av}QE[p]E[e_2]}{E[A]} + \frac{nC_iQ}{E[A]} + \frac{nc_{\alpha\beta}QE[p]E[e_2]}{E[A]} - \frac{nh_bB_3Q}{D} +$$

$$\frac{h_v}{2P}\left[(2n - n^2)Q^2 + \frac{n(n-1)PQ^2(1-E[p])(1-E[e_1])}{E[A]D} + \frac{n^2Q^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right] + \frac{h_bQ^2}{2D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right] \qquad (3.19)$$

After solving and applying theory of expectation the equation 3.16 becomes

$$E[TC_C(n,Q,B_3)] = S_v + K + nF + \frac{n(b+h_b)B_3^2}{2D} + \frac{nc_wQE[p]}{E[A]} + \frac{nC_rQ(1-E[p])E[e_1]}{E[A]} + \frac{nC_{av}QE[p]E[e_2]}{E[A]} + \frac{nC_iQ}{E[A]} + \frac{nc_{\alpha\beta}QE[p]E[e_2]}{E[A]} - \frac{nh_bB_3Q}{D} +$$

$$\frac{h_v}{2P}\left[\left(2n-\ n^2\right)Q^2+\ \frac{n(n-1)PQ^2(1-E[p])(1-E[e_1])}{E[A]D}\ +\ \frac{n^2Q^2\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A^2]}\right]+$$

$$\frac{h_bQ^2}{2D}\left[n+\ \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right] \qquad (3.20)$$

As explained below in Para 3.6 of this thesis, The Renewal and Reward Theorem is used to find costs for an infinite duration when cost or reward for cost of a single cycle is known.

$$Cost\ or\ Reward\ for\ infinite\ duration=\ \frac{Cost\ or\ Reward\ of\ one\ cycle}{Duration\ of\ the\ cycle}$$

Applying the Renewal and Reward Theorem to equation 3.17 we get expected total cost ETC(n,Q).

$$ETC(n,Q)=\ \frac{E[TC_c(n,Q)]}{E[T_c]}$$

$$ETC(n,Q)=$$

$$[S_v+K+nF\ ]*\frac{DE[A]}{n(1-E[p])(1-E[e_1])Q}+\left[nc_wE[p]+nC_r(1-E[p])E[e_1]+nC_{av}E[p]E[e_2]+nC_i+\right.$$

$$\left.nc_{\alpha\beta}E[p]E[e_2]\right]*\frac{D}{n(1-E[p])(1-E[e_1])}+$$

$$\left[\frac{h_v}{2P}\left\{\left(2n-n^2\right)+\frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]}+\frac{n^2\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E\left[A^2\right]}\right\}+\right.$$

$$\left.\frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\ (1-E[p])(1-E[e_1])}{2E[A^2]D}\right]Q*\ \frac{DE[A]}{n(1-E[p])(1-E[e_1])}$$

Solving it we got formula for the optimal value for Q. The formula is given below

$$Q*=$$

$$\sqrt{\frac{[S_v+K+nF\ ]}{\frac{h_v}{2P}\left\{(2n-n^2)+\frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]}+\frac{n^2\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A^2]}\right\}+\ \frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\ (1-E[p])(1-E[e_1])}{2E[A^2]D}}}$$

Applying the Renewal and Reward Theorem to equation 3.18 we get expected total cost

$$ETC_b(Q)=\frac{E[TC_b(Q)]}{E[T]}$$

$$ETC_b(Q) = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{c_{\alpha\beta}DE[p]E[e_2]}{(1-E[p])(1-E[e_1])} + \frac{h_b}{2}\frac{[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]Q}{E[A]}$$

Solving it we got formula for the optimal value for Q. The formula is given below

$$Q^* = \sqrt{\frac{2(K+F)DE[A^2]}{h_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}](1-E[p])(1-E[e_1])}}$$

Applying the Renewal and Reward Theorem to equation 3.19 we get expected total cost ETC(n,Q).

$$ETC(n,Q) = \frac{E[TC_c(n,Q)]}{E[T_c]}$$

$$ETC(n,Q,B_3) = \left[S_v + K + nF + \frac{n(b+h_b)B_3^2}{2D}\right] * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])} + \Big[nc_w E[p] + n(1-C_r E[p])E[e_1] + nC_{av}E[p]E[e_2] + nC_i + nc_{\alpha\beta}E[p]E[e_2] - \frac{nh_b B_3 E[A]}{D}\Big] * \frac{D}{n(1-E[p])(1-E[e_1])} + \frac{DQE[A]}{2n(1-E[p])(1-E[e_1])}\left[\frac{h_v}{P}\left[(2n - n^2) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right] + \frac{h_b}{D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right]\right]$$

Solving it we got formula for the optimal value for Q. The formula is given below

$$Q^* = \sqrt{\frac{[S_v+K+nF]\,D}{\frac{h_v D}{2P}\left[(2n - n^2)+\frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A^2]}\right] + \frac{h_b}{2}\left[n+\frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right] - \frac{nh_b{}^2}{2(b+h_b)}}}$$

Applying the Renewal and Reward Theorem to equation 3.20 we get expected total cost

$$ETC_b(Q,B_3) = \frac{E[TC_b(Q,B_3)]}{E[T_c]}$$

$$ETC_b(Q,B_3) = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{C_{\alpha\beta}DE[p]E[e_2]E[A]}{(1-E[p])(1-E[e_1])} + \frac{bB_3^2E[A]}{2(1-E[p])(1-E[e_1])Q} + \frac{h_b E[A]Q}{2(1-E[p])(1-E[e_1])} - \frac{h_b B_3 E[A]}{(1-E[p])(1-E[e_1])} + \frac{h_b B_3{}^2 E[A]}{2(1-E[p])(1-E[e_1])Q} + \frac{h_b E[p]E[e_2]Q}{2E[A]}$$

Solving it we got formula for the optimal value for Q. The formula is given below

$$Q^* = \sqrt{\frac{2(K+F)D}{h_b\left[1+\frac{\mathrm{b}}{(\mathrm{b}+h_b)}+\frac{E[p]E[e_2](1-E[p])(1-E[e_1])}{E[A^2]}\right]}}$$

3.3.2 Studying model

The outcome of a model can be explained in two ways: qualitative and quantitative. Qualitative outcome explains the behaviour of stochastic models in details and deals "how" things will be done. Whereas quantitative outcome explains model behaviour in term of figures and explain the details in "how much".

Sensitivity analysis of the model is done to check how the model is sensitive to its input parameters. It is done by providing a range of input for an input parameter and examines the changes in model outcome. Sensitivity analysis has been performed on mathematical models of this thesis.

3.3.3 Testing model

The model developed is validated by testing. Following are various steps of mathematical model testing:

i) Testing the assumptions
ii) Testing Model structure
iii) Testing predictions of the model
iv) Estimating model parameters
v) Comparing two models of the same system

Assumptions taken in the model are a primary building block of a model. All assumptions which were taken into consideration for transforming verbal concepts into assumption and later into mathematical equations are rechecked for correctness. Mathematical equations, structure, parameters and expected predictions of the model are also rejecting for correctness. Predictions are checked with fresh data which was not used at the time of building of the model.

If there is another model already exists for similar system, then two models are compared to check how the current model will be used in future. The objective of the comparison is to examine what is the difference between two, how much the current model is effective and other relevant parameters. In this research work model developed is compared to similar type of existing models. Result of developmental model is better than existing models.

3.3.4 Using model

After development of a model, all details of the model must be explained in detail to end users. All model output should provide the precision of its estimate in term of standard error present in the output. For example, growth of animals, feed with same food each day, may not be the same. Some grow faster and some slower. So model must take consideration of such deviations.

The output of the model must be helpful for use in decision making and provide economical solutions to improve profitability.

Inventories of raw materials, intermediate product and finished products are maintained at different levels of the chain.

Maintaining Inventory by manufacturers, wholesalers and retailers are necessary for any company in the real world that is dealing with physical products. Manufacturers maintain inventories of raw materials for production and finished products waiting for shipment. Wholesalers and retailers maintain inventories of finished products so that they are available to be purchased by customers

The annual cost associated with maintaining inventories is very large and sometime it may reach to a quarter of the total values of items stored in an inventory. This cost added to the price of the product and resulted in the high price of the product. Reduction of the cost of inventory maintenance, reduce overall cost of products for customers. Reduction of volume of products needed to be stored an inventory reduce inventory cost. This needs proper planning and scheduling for production of products, quantity of items in shipment, number of shipments, etc. some Japanese companies (Toyota) introduced just-in-time inventory system that plan and schedule such that the needed items arrive at the time when the item is needed to be used. It reduced the need for an item to be stored in an inventory and inventory level reached almost zero.

3.4 Concave and Convex function

Convex or concave functions of a single variable

Convex function: A function of a single variable *f(x)* is a convex function if, for each pair of x, say x' and x'' (x' <x''),

$$f[\lambda x'' + (1-\lambda)x'] \leq \lambda f(x'') + (1-\lambda) f(x') \qquad (3.21)$$

for all value of λ such that $0 < \lambda < 1$. It is strictly convex function if $\leq$ is replaced by $<$. A convex function has one or more minimum values (local minimum values) whereas the strictly convex function has only one minimum value, (Hillier, 2012).

Concave function: A function of a single variable *f(x)* is a concave function if, for each pair of x, say x' and x'' (x' <x''),

$$f[\lambda x'' + (1-\lambda)x'] \geq \lambda f(x'') + (1-\lambda) f(x') \qquad (3.22)$$

for all value of λ such that $0 < \lambda < 1$. It is strictly convex function if ≥ is replaced by >. A concave function has one or more maximum values (local maximum values) whereas the strictly concave function has only one maximum value, (Hillier, 2012).

Convexity test for a function of a single variable: As per (Hillier, 2012), any function of a single variable $f(x)$ that possesses a second derivative then for all possible values of x, $f(x)$ is

1. Convex if and only if $\frac{d^2 f(x)}{dx^2} \geq 0$ for all possible values of x.
2. Strictly convex if and only if $\frac{d^2 f(x)}{dx^2} > 0$ for all possible values of x.
3. Concave if and only if $\frac{d^2 f(x)}{dx^2} \leq 0$ for all possible values of x.
4. Strictly Concave if and only if $\frac{d^2 f(x)}{dx^2} > 0$ for all possible values of x.

Convexity test for a function of two variables: When there are two variables, then test for convexity is shown in following table

Table 3.2 Convexity Test

Quantity	**Convex**	**Strictly Convex**	**Concave**	**Strictly Concave**
$\frac{\delta^2 f(x_1,x_2)}{\delta x_1^2} \frac{\delta^2 f(x_1,x_2)}{\delta x_2^2} - \left[\frac{\delta^2 f(x_1,x_2)}{\delta x_1 \delta x_2}\right]^2$	≥ 0	> 0	≥ 0	> 0
$\frac{\delta^2 f(x_1,x_2)}{\delta x_1^2}$	≥ 0	> 0	≤ 0	< 0
$\frac{\delta^2 f(x_1,x_2)}{\delta x_2^2}$	≥ 0	> 0	≤ 0	< 0
Values of (x_1, x_2)	All possible values			

(Source: (Hillier, 2012))

3.5 Theory of Expectation

As per (Kapur & Saxena, 1997), the expectation is the value, on average, of a random variable (or function of a random variable). The expectation in a probabilistic sense always averages over the possible values weighting by the probability of observing each value. The form of an expectation in the discrete case is particularly simple.

The expectation of a continuous random variable is defined as $E[X] = \int_{-\infty}^{\infty} xf(x)dx$ exists if and only if $E[X] = \int_{-\infty}^{\infty} |x| f(x)dx < \infty$

For continuous random variables, expectations may not exist if the probability of observing an arbitrarily large value (in the absolute sense) is very high. For example, in a Student's t distribution when the degree of freedom parameter υ is 1 (also known as a Cauchy distribution), the probability of observing a value with size |x| is proportional to x^{-1} for large x (in other words, $f(x) \propto c\, x^{-1}$) so that $x\, f(x) \approx c$ for large x. The range is unbounded, and so the integral of a constant, even if very small, will not converge, and so the expectation does not exist. On the other hand, when a random variable is bounded, its expectation always exists.

The expectations operator has a number of simple and useful properties:

- If c is a constant, then E [c] = c. This property follows since the expectation is an integral against a probability density which integrates to unity.
- If c is a constant, then E [c X] = c E [X]. This property follows directly from passing the constant out of the integral in the definition of the expectation operator.
- The expectation of the sum is the sum of the expectations,

$$E\left[\sum_{i=1}^{k} g_i(X)\right] = \sum_{i=1}^{k} E[g_i(X)] \qquad (3.23)$$

This property follows directly from the distributive property of multiplication.

- If "a" is a constant, then E [a + X] = a + E [X]. This property also follows from the distributive property of multiplication.

- E [f (X)] = f (E [X]) when f (x) is affine (i.e. f (x) = a + b x where “a” and “b” are constants). For general nonlinear functions, it is usually the case that E [f (X)] ≠ f (E [X]) when X is non-degenerate.
- E [X p] ≠ E [X]p except when p = 1 when X is non-degenerate, University of Auckland, New Zealand.

3.6 Renewal and Reward Theorem

There are many real life situations where renewals have come after a certain time interval. These renewals carry some rewards or costs. For example, an employee has a bank account where he/she gets a salary. He/she spends money from the bank account and the account gets renewal in the form of salary amount each month. Salary is his/her reward. Here time duration and salary amount are almost fixed. Similarly, an inventory gets renewal in the form of fresh lot of items at a time interval and there occurred a cost for items received. Here number of items in the lot and the time interval is not fixed and can be assumed as a random number with known probability distribution.

For these types of cases the renewal and reward theorem is used to get the expected reward/cost per cycle. As per (Tijms, 2003), the average reward/cost per unit time is equal to the expected reward/cost during one cycle divided by the expected length of one cycle

$$\lim_{t \to \propto} \frac{R(t)}{t} = \frac{E[R_1]}{E[C_1]} \tag{3.24}$$

Here t is time, $R(t)$ is reward for time t, $E[R_1]$ is expected reward for cycle 1 and $E[C_1]$ is expected length for cycle 1

CHAPTER 4

An integrated single-vendor, single-buyer inventory model for imperfect quality production, imperfect inspection at vendor site

4.1 Introduction

(Salameh & Jaber, 2000) had pointed out that items received by buyer in traditional production/inventory models were not of perfect quality. They had extended traditional economical order quantity (EOQ) model by taking account of defective items received by a vendor with a known probability density function. After receiving a lot, the vendor conducted 100% inspection process at a rate of x unit per unit time where x > D (the demand rate) before selling them at market. During the inspection process, Items classified as defective were kept in inventory and sold-out at discounted price at the end of each lot cycle. (Wee et al., 2007) extended (Salameh & Jaber, 2000) EOQ model and allowed shortage backordering by adding an assumption that customers were willing to wait till the next supply to arrive, whenever there was shortage of items. (Maddah & Jaber, 2008) applied renewal and reward theorem to rectify a flaw in (Salameh & Jaber, 2000) EOQ model to get an exact expression for expected profit. (Khan et al., 2011) stated that the inspection process was itself imperfect. Items could be classified wrong way. For example, defective items could be classified as non-defective and non-defective as defective. Defective items sold (due to inspection error) in the market were returned back by customers for replacement with fresh items and stored in the inventory. At the end of each inspection process, all defective items returned back by customers along with items identified defective during the inspection process were returned back to the vendor for disposal. Classifying non-defective items as defective also caused loss of profit because non-defective items were sold at discounted price. Because of inspection errors, inventory level increased and extra demand of items was created. (Hsu & Hsu, 2012), through a technical note corrected an assumption of (Wee et al., 2007) that all, backorder were cleared immediately as soon as a fresh lot of the items arrived, ignoring the time required for inspection of items.

Above research works assumed that inspection of items was done at buyer site which was first proposed by (Salameh & Jaber, 2000). Other research works after (Salameh & Jaber, 2000) followed the same assumption. The assumption has been changed by assuming that the vendor will do inspection of items along with production of items in this research work.

4.2 Notation and Assumptions for the Mathematical Model

Q_P	: the count of items of a lot that are produced per production cycle
Q	: the count of items of a lot that delivered from the vendor to the buyer
n	:the number of deliveries to the buyer per production cycle, a positive integer $(Q_P = nQ)$
D	:the demand rate of items per year
P	:the production rate of items (P > D)
x	:the items inspection rate of items x > P
S_v	:the setup cost per production cycle
K	:the ordering cost per order for the buyer
C_i	:the vendor's inspection cost per unit
C_w	:the vendor's cost per unit for producing defective items (warranty cost)
$C_{\alpha\beta}$	:the buyer's cost per unit for selling defective items in the market (due to Type II Error)
C_{av}	:the vendor's cost of selling defective items in the market (due to Type II Error)
C_r	:the vendor's cost for rejecting non-defective items as defective items
h_v	:the inventory holding cost per unit item of vendors

h_b	:the inventory holding cost per unit item for buyer
F	:the transportation cost per delivery of items
p	:the probability of production of defective items
f (p)	:the probability density function of p
e_1	:the probability of type I inspection Error (Classifying Non-Defective item as Defective items)
f (e_1)	:the probability density function of e_1
e_2	:the probability of type II inspection Error (Classifying Defective item as Non-Defective items)
f (e_2)	:the probability density function of e_2
B_1	:the number of items classified as defective after inspection per production lot
B_2	:the number of defective items classified as Non-Defective item per production lot (Type II Inspection Error)
T	:the time interval between two successive deliveries to buyers of Q items
T_1	:the time period during which vendor produce items
T_2	:the time period during which vendor supplies items from inventory
T_C	:the production cycle time (T_C = nT)
*	:the superscript to represent optimal value

In this single-vendor and single-buyer model, it is assumed that items are produced by the vendor. The rate of production *P* of items for production unit of manufacturing industries is greater than the demand rate *D* of the items. Items are produced in one production setup cycle and sent to the buyer in *n* multiple lots after their inspection. The production process of manufacturing industries have machine errors and human errors and is of imperfect quality and can produce some defective items with probability *p* with a known probability distribution *f (p)*. Before sending items in lots to buyer, the inspection of all items is performed to filter out B_1

defective items. To save time, the inspection is done along with production. The inspection rate x is greater than the production rate P of items ($x > P$) so that items that produced will be inspected without any delay in manufacturing industries. The inspection process, due to human factors, is also imperfect and can have following two types of errors

Type I : Inspector incorrectly classifies non-defective items as defective items with probability e_1 and probability density function $f(e_1)$

Type II : Inspector incorrectly classifies defective items as non-defective items with probability e_2 and probability density function $f(e_2)$

When, type I error occurs, an inspector classifies a non-defective item as defective item. It leads to loss of C_j per unit item of revenue as some non-defective (classified as defective) items are being disposed at a discounted rate along with other defective items.

When, type II error occurs, an inspector classifies a defective item as non-defective item. These types of B_2 items are sent to buyer to sell at market in each lot. Defects of such items are identified by customers during their use. Customers approach the buyer to get the defective item replaced by a fresh item for its warranty. The buyer keeps B_2 defective items, received from customers, in its inventory and returned back to the vendor at the end of each lot cycle. The vendor then disposed these B_2items immediately after receipt from the buyer at discounted rate. The cost to the buyer is $C_{\alpha\beta}$ and to the vendor is C_{av} for selling per unit defective item.

4.3 Mathematical Model

Figure 4.1

Vendor's Inventory

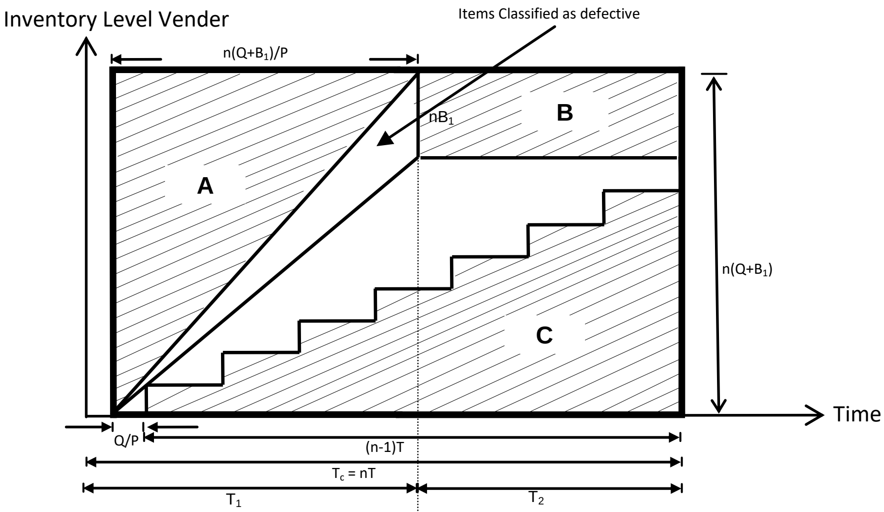
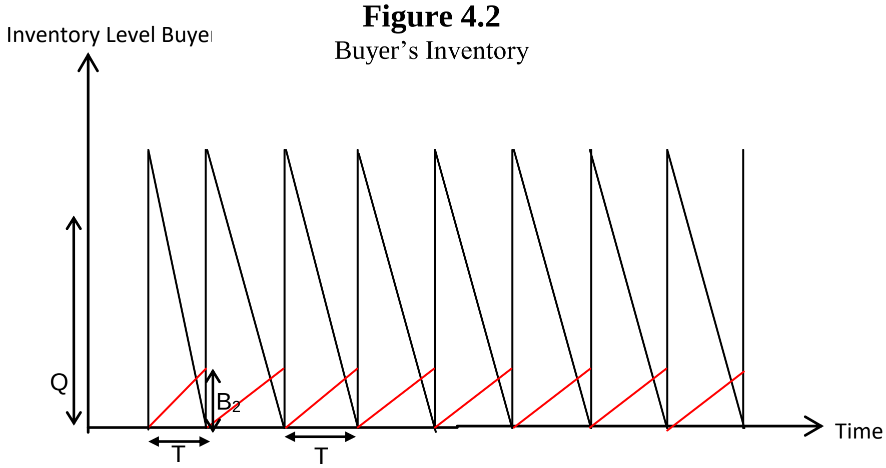


Figure 4.2

Buyer's Inventory



Number of defective items produced in a production lot $\mathbf{Q + B_1}$ with its probability distribution $\mathbf{p}$ is

$$= (Q + B_1)p \tag{4.1}$$

Number of non-defective items produced in a production lot $\mathbf{Q + B_1}$ with probability distribution $\mathbf{p}$ to produce defective items is

$$= (Q + B_1)(1 - p) \tag{4.2}$$

Number of items classified as defective which are actually non-defective due to Type I inspection Error $\mathbf{e_1}$ in a production lot $\mathbf{Q + B_1}$, probability distribution $\mathbf{p}$ to produce defective items in production is

(Non-Defective → Defective)

$$= (Q + B_1)(1 - p)e_1 \tag{4.3}$$

Number of items classified as non-defective which are actually defective due to Type II inspection Error $\mathbf{e_2}$ in a production lot $\mathbf{Q + B_1}$, probability distribution $\mathbf{p}$ to produce defective items in production is

(Defective → Non-Defective)

$$= (Q + B_1)pe_2 \tag{4.4}$$

Number of items classified as defective which are actually defective, considering Type I inspection Error $\mathbf{e_1}$ and Type II inspection Error $\mathbf{e_2}$ in a production lot $\mathbf{Q + B_1}$, probability distribution $\mathbf{p}$ to produce defective items in production is

(Defective → Defective)

$$= (Q + B_1)p(1 - e_2) \tag{4.5}$$

Number of items classified as non-defective which are actually non-defective, considering Type I inspection Error $\mathbf{e_1}$ and Type II inspection Error $\mathbf{e_2}$ in a production lot $\mathbf{Q + B_1}$, probability distribution $\mathbf{p}$ to produce defective items in production is

(Non-Defective → Non-Defective)

$$= (Q + B_1)\{(1-p)(1-e_1)\} \tag{4.6}$$

B_1 is the number of items that are being classified as defective items, due to type I inspection error. B_1items are produced additional to the Q items by the vendor to enable him to supply Q items to buyer at the beginning of each replenishment cycle. Thus, for each supply of Q items the vendor produced $(Q + B_1)$ items.

Thus B_1= Defective items classified as defective + Type I error (classify non-defective as defective)

$$B_1 = (Q + B_1)p(1 - e_2) + (Q + B_1)(1 - p)e_1 \tag{4.7}$$

after solving it,

$$B_1 = \frac{p(1-e_2) + (1-p)e_1}{1-\{p(1-e_2) + (1-p)e_1\}} Q \tag{4.8}$$

B_2 items are defective items due to type II Error. (Classify defective as non-defective)

$$B_2 = (Q+B_1)pe_2 \tag{4.9}$$

Putting the value of B_1 and solving

$$B_2 = \frac{pe_2 Q}{1-\{p(1-e_2)+(1-p)e_1\}} \tag{4.10}$$

Defective items, B_2 are sold in the market and replaced by consumers with fresh items. Replacements of defective items create additional demand of fresh items. The effective demand D' includes both real market demand D and replacement demands$\left(\frac{B_2}{T}\right)$. Thus, effective demand $D' = D + \frac{B_2}{T}$.

The cycle length of each delivery T for the buyer is $T = \frac{Q}{D'}$. Substituting value of D'

$$T = \frac{Q}{D + \frac{B_2}{T}} = \frac{QT}{DT + B_2} \quad i.e., \; DT + B_2 = Q \;\; i.e., T = \frac{Q - B_2}{D}$$

Solving above,

$$T = \frac{Q-B_2}{D}$$

$$\mathrm{T} = \left[\mathrm{Q} - \frac{\mathrm{pe_2Q}}{1\text{-}\{\mathrm{p}(1\text{-}\mathrm{e_2})+(1\text{-}\mathrm{p})\mathrm{e_1}\}}\right]\frac{1}{\mathrm{D}}$$

$$T = \frac{(1-p)(1-e_1)Q}{[1-\{p(1-e_2)+(1-p)e_1\}]D}$$

$$T = \frac{Q-B_2}{D}$$

$$\mathrm{T} = \left[\mathrm{Q} - \frac{\mathrm{pe_2Q}}{1\text{-}\{\mathrm{p}(1\text{-}\mathrm{e_2})+(1\text{-}\mathrm{p})\mathrm{e_1}\}}\right]\frac{1}{\mathrm{D}} \qquad (4.11)$$

4.4 The Buyer's cost per production cycle

The buyer gets Q items at the beginning of each replenishment cycle. Defective items B_2 (replaced by customers and stored in buyer's inventory) is sent to the vendor at the end of each cycle for disposal at discounted rate. The inventory holding cost of the buyer is

HC_b = n * (Holding Cost of Q items for T time + Holding Cost of B_2 items for T time)

$$HC_b = n * (h_b \frac{Q}{2} T + h_b \frac{B_2}{2} T)$$

$$HC_b = \frac{nh_b}{2}(Q + B_2)T$$

Here $T = \frac{Q}{D'}$

Substituting values of B_2, and T and solving, holding cost to the buyer is

$$HC_b = \frac{nh_b}{2}\left(Q + \frac{\mathrm{pe_2}Q}{1\text{-}\{\mathrm{p}(1\text{-}\mathrm{e_2})+(1\text{-}\mathrm{p})\mathrm{e_1}\}}\right) * \frac{(1-p)(1-e_1)Q}{[1-\{p(1-e_2)+(1-p)e_1\}]D}$$

$$HC_b = \frac{nh_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{[1-\{p(1-e_2)+(1-p)e_1\}]^2}Q^2$$

The total cost to the buyer per production cycle includes ordering cost, transportation cost, post-sale failure cost (due to sales of defective items) and holding cost. These costs will depend on three variable parameters, i.e., *n* (number of orders per production cycle) and *Q* (lot size). Thus, the total cost per production cycle is

$$TC_b(n,Q) = K + nF + nc_{\alpha\beta}(Q+B_1)pe_2 + \frac{nh_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{[1-\{p(1-e_2)+(1-p)e_1\}]^2}Q^2$$

$$TC_b(n,Q) =$$

$$K + nF + \frac{nc_{\alpha\beta}pe_2\,Q}{1-\{p(1-e_2)+(1-p)e_1\}} + \frac{nh_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{[1-\{p(1-e_2)+(1-p)e_1\}]^2}Q^2$$

Let

$$A = 1 - \{p(1-e_2) + (1-p)e_1\} \tag{4.12}$$

then

$$TC_b(n,Q) = K + nF + \frac{nc_{\alpha\beta}pe_2\,Q}{A} + \frac{nh_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{A^2}Q^2 \tag{4.13}$$

4.5 The Vendor's cost per production cycle

At the vendor's site, inspection test is conducted immediately after production of items to classify them as defective or non-defective. After inspection, Q non-defective items are sent to the buyer in n lots at an equal interval of time *T*. Surplus items are kept at vendor's inventory for future supply.

Items, classified as defective, are also kept in inventory. When production is over and all defective items are sold at discounted price. In figure nB_1 shows inventory of defective items

Figure 4.1 shows that vendor's holding cost per production cycle can be obtained as (see, for example, (Goyal et al., 2003), (Huang, 2004) and (Hsu & Hsu, 2012b))

Inventory holding cost per cycle = h_v [bold area – shaded area A, B and C]

$= h_v\ [\ n(Q+B_1)\left\{\frac{Q}{P} + (n-1)T\right\} - \frac{1}{2}n(Q+B_1)\ \frac{n(Q+B_1)}{P} - \left\{\frac{Q}{P} + (n-1)T - \frac{n(Q+B_1)}{p}\right\} nB_1 - \frac{n(n-1)TQ}{2}\ ]$

$= h_v\ [\ \frac{n(Q+B_1)}{P}\{Q + (n-1)PT\} - \frac{n^2(Q+B_1)^2}{2P} - \frac{nB_1}{P}\{Q + (n-1)PT - n(Q+B_1)\} - \frac{n(n-1)TQ}{2}\ ]$

$= \frac{h_v}{2P}\ [\ 2n(Q +$

$B_1)\ \{Q + (n-1)PT\} - n^2(Q + B_1)^2 - 2nB_1\ \{Q + (n-1)PT - n(Q+B_1)\} - n(n-1)PTQ\ ]$

$= \frac{h_v}{2P}\ [\ \{2n(Q + B_1)Q + 2(n-1)n(Q + B_1)PT\} - n^2(Q^2 + 2B_1Q +$

$B_1{}^2) - \{2nB_1Q + 2n(n-1)B_1PT - 2n^2B_1(Q+B_1)\} - n(n-1)PTQ\ ]$

$= \frac{h_v}{2P}\ [\ 2nQ^2 + 2nB_1Q + 2n(n-1)PTQ + 2n(n-1)B_1PT - n^2Q^2 - 2n^2B_1Q -$

$n^2B_1{}^2 - 2nB_1Q - 2n(n-1)B_1PT + 2n^2B_1(Q+B_1) - n(n-1)PTQ\ ]$

$= \frac{h_v}{2P}\ [\ 2nQ^2 + 2nB_1Q + 2n(n-1)PTQ + 2n(n-1)B_1PT - n^2Q^2 - 2n^2B_1Q -$

$n^2B_1{}^2 - 2nB_1Q - 2n(n-1)B_1PT + 2n^2B_1Q + 2n^2B_1{}^2 - n(n-1)PTQ\ ]$

$= \frac{h_v}{2P}\ [\ 2nQ^2 + 2nB_1Q - 2nB_1Q + 2n(n-1)PTQ + 2n(n-1)B_1PT - 2n(n-1)B_1PT -$

$n^2Q^2 - 2n^2B_1Q + 2n^2B_1Q - n^2B_1{}^2 + 2n^2B_1{}^2 - n(n-1)PTQ\ ]$

$= \frac{h_v}{2P}\ [\ 2nQ^2 + 2n(n-1)PTQ - n^2Q^2 - n^2B_1{}^2 + 2n^2B_1{}^2 - n(n-1)PTQ\ ]$

$= \frac{h_v}{2P}\ [\ 2nQ^2 - n^2Q^2 + 2n(n-1)PTQ - n(n-1)PTQ - n^2B_1{}^2 + 2n^2B_1{}^2]$

$= \frac{h_v}{2P}\ [\ (2n - n^2)Q^2 + n(n-1)PTQ + n^2B_1{}^2]$

Replacing the values of T and B_1

$= \frac{h_v}{2P}\left[\ (2n - n^2)Q^2 + n(n-1)PQ\ \frac{(1-p)(1-e_1)Q}{[\ 1-\{p(1-e_2)+(1-p)e_1\}]D} + n^2\,\frac{\{p(1-e_2)+(1-p)e_1\}^2Q^2}{[1-\{p(1-e_2)+(1-p)e_1\}]^2}\right]$

$= \frac{h_v}{2P}\left[\ (2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{[\ 1-\{p(1-e_2)+(1-p)e_1\}]D} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{[1-\{p(1-e_2)+(1-p)e_1\}]^2}\right]$

From Equation (10) $\ A = 1 - \{p(1 - e_2) + (1 - p)e_1\}$, putting the value of A in above

Inventory holding cost per cycle

$$= \frac{h_v}{2P}\left[\left(2n - n^2\right)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\right]$$

Setup Cost = S_v

Warranty Cost = $n(Q + B_1)pC_w$ $= \frac{npc_wQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{npc_wQ}{A}$

Type I Error = $n(Q + B_1)(1 - p)e_1C_r$ $= \frac{n(1-p)e_1C_rQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{n(1-p)e_1C_rQ}{A}$

Type II Error = $n(Q + B_1)pe_2C_{av}$ $= \frac{npe_2C_{av}Q}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{npe_2C_{av}Q}{A}$

Inspection Cost = $n(Q + B_1)C_i$ $= \frac{nC_iQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{nC_iQ}{A}$

Adding the cost of setup, warranty, Type I errors, Type II errors and inventory holding, the vendor's total cost per production cycle $TC_v(n,Q)$ for the vendor is

$$TC_v(n,Q) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\left[\left(2n - n^2\right)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\right] \tag{4.14}$$

4.6 The integrated model for vendor-buyer

The total cost of the integrated model of the vendor-buyer per production cycle is

$$TC_C(n,Q) = TC_v(n,Q) + TC_b(n,Q)$$

$$TC_C(n,Q) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\left[\left(2n - n^2\right)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\right] + K + nF + \frac{nc_{\alpha\beta}pe_2Q}{A} + \frac{nh_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}](1-p)(1-e_1)}{A^2}Q^2$$

$$TC_C(n,Q) = S_v + K + nF + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{nc_{\alpha\beta}pe_2Q}{A} + \frac{h_v}{2P}\left[\left(2n - n^2\right)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\right] + \frac{nh_b[1-\{p(1-2e_2)+(1-p)e_1\}](1-p)(1-e_1)Q^2}{2A^2D}$$

$$TC_C(n,Q) = S_v + K + nF + \left[npc_w + n(1-p)e_1C_r + npe_2C_{av} + nC_i + nc_{\alpha\beta}pe_2\right]\frac{Q}{A} +$$

$$\left[\frac{h_v}{2P}\left\{(2n - n^2) + \frac{n(n-1)(1-p)(1-e_1)P}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2}{A^2}\right\} + \frac{nh_b[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{2A^2D}\right]Q^2 \quad (4.15)$$

The expected total cost of the integrated model is

$$E[TC_C(n,Q)] = S_v + K + nF + \left[nc_wE[p] + nC_r(1-E[p])E[e_1] + nC_{av}E[p]E[e_2] + nC_i + nc_{\alpha\beta}E[p]E[e_2]\right]\frac{Q}{E[A]} +$$

$$\left[\frac{h_v}{2P}\left\{(2n - n^2) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{E[A]D} + \frac{n^2\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A^2]}\right\} + \frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}\right]Q^2$$

Where

$$E[A] = 1 - \{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}$$

Since the cycle time $T_c = \frac{n(1-p)(1-e_1)Q}{[1-\{p(1-e_2)+(1-p)e_1\}]D} = \frac{n(1-p)(1-e_1)Q}{AD}$,

Expected cycle time $E[T_C]$ is

$$E[T_c] = \frac{n(1-E[p])(1-E[e_1])Q}{DE[A]} \quad (4.16)$$

4.6.1 Application of Renewal and Reward theorem

Using the renewal and reward theorem, the expected total cost $ETC(n,Q,B_3)$ of the integrated model for vendor and buyer is

$$ETC(n,Q) = \frac{E[TC_C(n,Q)]}{E[T_c]}$$

$$ETC(n,Q) = [S_v + K + nF] * \frac{DE[A]}{n(1-E[p])(1-E[e_1])Q} + \left[nc_wE[p] + nC_r(1-E[p])E[e_1] + nC_{av}E[p]E[e_2] + nC_i + nc_{\alpha\beta}E[p]E[e_2]\right]\frac{Q}{E[A]} * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])} + \left[\frac{h_v}{2P}\left\{(2n - n^2) +\right.\right.$$

$$\frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\Bigg\} +$$

$$\frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}\Bigg] Q^2 * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])}$$

$ETC(n, Q) =$

$$[S_v + K + nF\ ] * \frac{DE[A]}{n(1-E[p])(1-E[e_1])Q} + \Big[nc_w E[p] + nC_r(1 - E[p])E[e_1] + nC_{av}E[p]E[e_2] + nC_i +$$

$$nc_{\alpha\beta}E[p]E[e_2]\Big] * \frac{D}{n(1-E[p])(1-E[e_1])} +$$

$$\left[\frac{h_v}{2P}\left\{\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E\left[A^2\right]}\right\} +\right.$$

$$\left.\frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}\right] Q * \frac{DE[A]}{n(1-E[p])(1-E[e_1])} \qquad (4.17)$$

4.6.2 Finding optimal solution

The first derivative of above with respect to Q

$$\frac{\partial ETC(n,Q)}{\partial Q} =$$

$$-[S_v + K + nF\ ] * \frac{DE[A]}{n(1-E[p])(1-E[e_1])Q^2} + 0 + \left[\frac{h_v}{2P}\left\{\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} +\right.\right.$$

$$\left.\left.\frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right\} + \frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}\right] *$$

$$\frac{DE[A]}{n(1-E[p])(1-E[e_1])}$$

$$\frac{\partial ETC(n,Q)}{\partial Q} =$$

$$-[S_v + K + nF\ ] * \frac{DE[A]}{n(1-E[p])(1-E[e_1])Q^2} + \left[\frac{h_v}{2P}\left\{\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} +\right.\right.$$

$$\left.\left.\frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right\} + \frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}\right] *$$

$$\frac{DE[A]}{n(1-E[p])(1-E[e_1])} \qquad (4.18)$$

The second derivative of the above equation with respect to Q

$$\frac{\partial^2 ETC(n,Q)}{\partial Q^2} = [S_v + K + nF\ ] * \frac{DE[A]}{n(1-E[p])(1-E[e_1])Q^3} - \qquad (4.19)$$

Where

$$E[A] = 1 - \{E[p](1 - E[e_2]) + (1 - E[p])E[e_1]\}$$

4.6.2.1 Testing convexity of the cost function

Values of $(1 - E[p])$, $(1 - E[e_1])$ and E[A] are positive because $0 < E[p] < 1$, $0 < E[e_1] < 1$, $0 < E[e_2] < 1$ and other variable are positive. This implies $\frac{\partial^2 ETC(n,Q)}{\partial Q^2} > 0$ indicating that curve for total cost for different values of Q is strictly convex and there exists a global (only one) minimum cost for a value of Q (say Q*) [Please refer 2.3 of methodology of research)]. By equating first derivative equal to zero, the value of the Q* can be obtained.

$$-[S_v + K + nF\,] * \frac{DE[A]}{n(1-E[p])(1-E[e_1])Q^2} + \left[\frac{h_v}{2P}\left\{\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right\} + \frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}\right] * \frac{DE[A]}{n(1-E[p])(1-E[e_1])} = 0$$

$$\left[\frac{h_v}{2P}\left\{\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right\} + \frac{nh_b\left[1-\left\{E[p]\left(1-2E[e_2]\right)+(1-E[p])E[e_1]\right\}\right]\,(1-E[p])(1-E[e_1])}{2E[A]^2D}\right] = [S_v + K + nF\,] * \frac{1}{Q^2}$$

$$Q^2 * \left[\frac{h_v}{2P}\left\{\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right\} + \frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}\right] = [S_v + K + nF\,]$$

$$Q^2 * =$$

$$\frac{[S_v + K + nF\,]}{\frac{h_v}{2P}\left\{\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right\} + \frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}}$$

4.6.3 The optimal solution of integrated model

$$Q^* = \sqrt{\frac{[S_v+K+nF\,]}{\frac{h_v}{2P}\left\{(2n-n^2)+\frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]}+\frac{n^2\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A^2]}\right\}+\frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}}} \tag{4.20}$$

Where $E[A] = 1 - \{E[p](1 - E[e_2]) + (1 - E[p])E[e_1]\}$

$A = 1 - \{p(1 - e_2) + (1 - p)e_1\}$

$A^2 = [1 - \{p(1 - e_2) + (1 - p)e_1\}]^2$

$= 1 - 2p + 2pe_2 - 2e_1 + 2pe_1 + p^2 - 2p^2e_2 + p^2{e_2}^2 + 2pe_1 - 2pe_1e_2 - 2p^2e_1 +$

$2p^2e_1e_2 + {e_1}^2 - 2p{e_1}^2 + p^2{e_1}^2$

and

$E[A^2] = 1 - 2E[p] + 2E[p]E[e_2] - 2E[e_1] + 2E[p]E[e_1] + E[p^2] - 2E[p^2]E[e_2] +$

$E[p^2]E[{e_2}^2] + 2E[p]E[e_1] - 2E[p]E[e_1]E[e_2] - 2E[p^2E[e_1] + 2E[p^2]E[e_1]E[e_2] +$

$E[{e_1}^2] - 2E[p]E[{e_1}^2] + E[p^2E[{e_1}^2]$ (4.21)

4.7 The independent buyer model

If the buyer and the vendor do not work in collaboration for maximizing benefits and reducing expected total cost, the buyer places orders and get ordered items in single lot. The vendor produces ordered items and sends them to the buyer after 100% inspection of items. In this case number of lots shipped per order becomes one.

The total cost to the buyer and the time duration will be

$$TC_b(Q) = K + F + \frac{c_{\alpha\beta}pe_2\,Q}{A} + \frac{h_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{A^2}Q^2$$

$T = \frac{(1-p)(1-e_1)Q}{AD}$

The expected total cost for the buyer and expected time duration are

$E[TC_b(Q)] =$

$$K + F + \frac{c_{\alpha\beta}E[p]E[e_2]Q}{[A]} + \frac{h_b}{2D}\frac{[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{E[A^2]}Q^2$$

$$E[T] = \frac{(1-E[p])(1-E[e_1])Q}{DE[A]}$$

Using renewal and Reward theorem [please refer 3.5 of methodology of research and equation 3.4]

$$ETC_b(Q) = \frac{E[TC_b(Q)]}{E[T]}$$

$ETC_b(Q) =$

$$\frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{c_{\alpha\beta}E[p]E[e_2]Q}{E[A]}\frac{DE[A]}{(1-E[p])(1-E[e_1])Q} +$$

$$\frac{h_b}{2D}\frac{[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])Q^2}{E[A^2]}\frac{DE[A]}{(1-E[p])(1-E[e_1])Q}$$

$$ETC_b(Q) = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{c_{\alpha\beta}DE[p]E[e_2]}{(1-E[p])(1-E[e_1])} + \frac{h_b}{2}\frac{[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]Q}{E[A]} \tag{4.22}$$

Taking First derivative of $ETC_b(Q)$ with respect to Q

$$\frac{\partial ETC_b(Q)}{\partial Q} = -\frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^2} + 0 + \frac{h_b}{2}\frac{[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]}{E[A]}$$

$$\frac{\partial ETC_b(Q)}{\partial Q} = -\frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^2} + \frac{h_b}{2}\frac{[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]}{E[A]}$$

Taking Second derivative of ETC_b (Q) with respect to Q

$$\frac{\partial^2 ETC_b(Q)}{\partial Q^2} = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^3}$$

The value of $\frac{\partial^2 ETC_b(Q)}{\partial Q^2}$ is always positive because $0 \le E[p] \le 1$, $0 \le E[e_1] \le 1$, $0 \le (1- E[p]) \le 1$, $0 \le (1-E[e_1]) \le 1$, $0 \le (1-E[A]) \le 1$ and values of K, F and Q are positive. This indicates that the curve $\frac{\partial^2 ETC_b(Q)}{\partial Q^2}$ is strictly convex and there exist a global (only one) minimum value [Please refer 3.3 of methodology of research)].

To get the minimum could be calculated by putting $\frac{\partial ETC_b(Q)}{\partial Q} = 0$.

$$-\frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^2} + \frac{h_b}{2}\frac{[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]}{E[A]} = 0$$

$$\frac{h_b}{2}\frac{[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]}{E[A]} = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^2}$$

$$Q^2 = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])}\,\frac{2E[A]}{h_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]}$$

4.7.1 The optimal solution for the independent buyer model

$$Q^* = \sqrt{\frac{2(K+F)DE[A^2]}{h_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}](1-E[p])(1-E[e_1])}} \tag{4.23}$$

The total cost of vendor is

$$TC_v(n,Q) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\Big[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\Big]$$

$$TC_v(Q) = S_v + \frac{c_w pQ}{A} + \frac{C_r(1-p)e_1Q}{A} + \frac{C_{av}pe_2Q}{A} + \frac{C_iQ}{A} + \frac{h_v}{2P}\Big[Q^2 + \frac{\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\Big]$$

$$E[TC_v(Q)] = S_v + \frac{c_wE[p]Q}{E[A]} + \frac{C_r(1-E[p])E[e_1]Q}{E[A]} + \frac{C_{av}E[p]E[e_2]Q}{E[A]} + \frac{C_iQ}{E[A]} + \frac{h_v}{2P}\Big[Q^2 + \frac{\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2Q^2}{E[A^2]}\Big]$$

$$E[TC_v(Q)] = S_v + \{c_wE[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\}\frac{Q}{E[A]} + \frac{h_v}{2P}\Big[1 + \frac{\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A^2]}\Big]Q^2$$

$$ETC_v(Q) = \frac{E[TC_v(Q)}{E[T]} \text{ where } E[T] = \frac{(1-E[p])(1-E[e_1])Q}{DE[A]}$$

$$ETC_v(Q) = S_v * \frac{DE[A]}{(1-E[p])(1-E[e_1])Q} + \{c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\} \frac{Q}{E[A]} * \frac{DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{h_v}{2P}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right] Q^2 * \frac{DE[A]}{(1-E[p])(1-E[e_1])Q}$$

$$ETC_v(Q) = \frac{S_v DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{D\{c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\}}{(1-E[p])(1-E[e_1])} + \frac{h_v D}{2P(1-E[p])(1-E[e_1])}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A]}\right] Q$$

$$ETC_v(Q) = \frac{D}{(1-E[p])(1-E[e_1])}\left[\frac{S_v E[A]}{Q} + c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i + \frac{h_v}{2P}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A]}\right] Q\right] \quad (4.24)$$

From equation (4.22) and (4.24) total cost will be

Total cost $ETC(Q) = ETC_b(Q) + ETC_v(Q)$

$$ETC(Q) = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{c_{\alpha\beta} DE[p]E[e_2]}{(1-E[p])(1-E[e_1])} + \frac{h_b}{2}\frac{[1-\{E[p](1-2E[e_2]) + (1-E[p])E[e_1]\}]Q}{E[A]} + \frac{S_v DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{D\{c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\}}{(1-E[p])(1-E[e_1])} + \frac{h_v D}{2P(1-E[p])(1-E[e_1])}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A]}\right] Q$$

$$ETC(Q) = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{S_v DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{c_{\alpha\beta} DE[p]E[e_2]}{(1-E[p])(1-E[e_1])} + \frac{D\{c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\}}{(1-E[p])(1-E[e_1])} + \frac{h_b}{2}\frac{[1-\{E[p](1-2E[e_2]) + (1-E[p])E[e_1]\}]Q}{E[A]} + \frac{h_v D}{2P(1-E[p])(1-E[e_1])}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A]}\right] Q$$

$$ETC(Q) = \frac{(K+F+S_v)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{D\{c_{\alpha\beta} E[p]E[e_2] + c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\}}{(1-E[p])(1-E[e_1])} + \frac{h_b}{2}\frac{[1-\{E[p](1-2E[e_2]) + (1-E[p])E[e_1]\}]Q}{E[A]} + \frac{h_v D}{2P(1-E[p])(1-E[e_1])}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A]}\right] Q \quad (4.25)$$

4.8 Numerical and sensitivity analysis

Considering the integrated vendor- buyer inventory system, where inspection is being performed at vendor site, following parameters are taken. These parameters are also used by (Salamesh & Jaber, 2000), (Wee at al., 2007), (Maddah & Jaber, 2008) and (Hsu & Hsu, 2012) for their numerical analysis and cross analysis of their results.

Production rate,	P	= 160,000 units/year
Demand rate,	D	=50,000 units/year
Inspection rate,	x	=175,200 units/year
Set up cost of vendor,	S_v	= $300/ production run
Ordering cost to the buyer,	K	= $100/ order
The holding cost for vender,	h_v	= $2/unit/year
Holding cost to the buyer,	h_b	= $5/unit/year
Freight (transportation) cost,	F	= $25/delivery
Inspection cost,	C_i	= $0.5/unit
The cost of producing a defective item,	C_w	= $50/unit
The cost of rejecting a non-defective item,	C_r	= $100/unit
The buyer's post-sales failure cost	$C_{\alpha\beta}$	= $200/unit
The vendor's post-sales failure cost	C_{av}	= $300/unit
The backordering cost	b	= $10/unit/year

The items defective percentages p during production and type I inspection error e_1 and type I inspection error e_2 follow a uniform distribution with

$$f(p) = \begin{cases} \frac{1}{\beta}, & 0 \le p \le 0 \\ 0, & otherwise \end{cases} \qquad f(e_1) = \begin{cases} \frac{1}{\lambda}, & 0 \le e_1 \le 0 \\ 0, & otherwise \end{cases} \qquad f(e_2) = \begin{cases} \frac{1}{\eta}, & 0 \le e_2 \le 0 \\ 0, & otherwise \end{cases}$$

Then

$$E[p] = \int_0^{\beta} p f(p) dp = \int_0^{\beta} \frac{p}{\beta} dp = \frac{\beta}{2} \qquad E[p^2] = \int_0^{\beta} p^2 f(p) dp = \int_0^{\beta} \frac{p^2}{\beta} dp = \frac{\beta^2}{3}$$

$E[e_1] = \int_0^{\lambda} e_1 f(e_1) de_1 = \int_0^{\lambda} \frac{p}{\beta} de_1 = \frac{\lambda}{2}$ $\quad E[e_1{}^2] = \int_0^{\lambda} e_1{}^2 f(p) de_1 = \int_0^{\lambda} \frac{e_1{}^2}{\beta} de_1 = \frac{\lambda^2}{3}$

$E[p] = \int_0^{\eta} e_2 f(e_2) de_2 = \int_0^{\eta} \frac{p}{\beta} de_2 = \frac{\eta}{2}$ $\quad E[e_2{}^2] = \int_0^{\eta} e_2{}^2 f(e_2) de_2 = \int_0^{\eta} \frac{p^2}{\beta} de_2 = \frac{\eta^2}{3}$

If it is assumed that β = λ = η = 0.04, then the expected total cost of the integrated single buyer and single vendor is a function of the number of lots "n" and number of items supplied Q per lot. For the above given numerical parameters, the optimal solution is n* = 7, Q* = 769.4 and total minimum expected cost is 201226.23 (See Table 4.1)

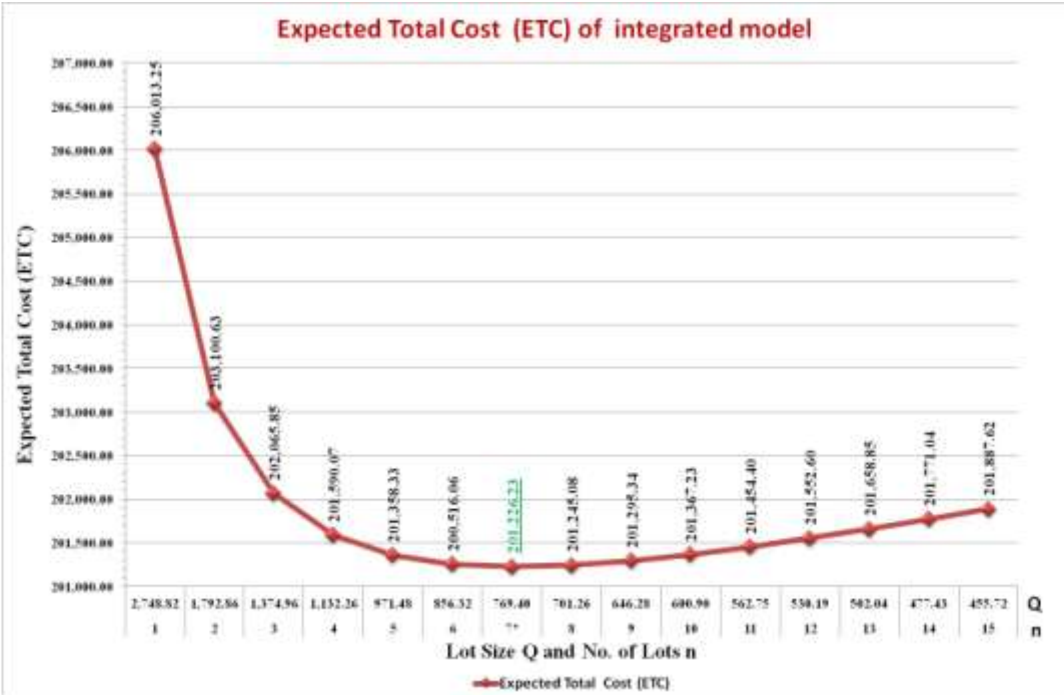
4.8.1 Minimum Expected Total Cost and its comparison

The minimum Expected Total Cost for the integrated solution with respect to n and Q where P=160,000, D=50,000, S_v=300, K=100, h_v=2, h_b=5, F=25, c_i=.0.5, c_w=50, c_r=100, $c_{\alpha\beta}$=200, c_{av}=300 and β = λ = η = 0.04 with comparison to result found by (Hsu and Hsu, 2012)

Table 4.1

	Result of Numerical Analysis			(Hsu & Hsu, 2012b) Result		
n	Q*(n)	No of Shipments to meet demand D	ETC(n, Q*(n))	Q*(n)	ETC(n,Q*(n))	Improvement in ETC(n,Q*(n))
1	2,748.82	18.19	206,013.25	2,817.49	206,251.90	238.65
2	1,792.86	27.89	203,100.63	1,839.47	203,281.73	181.10
3	1,374.96	36.36	202,065.85	1,411.65	202,224.24	158.39
4	1,132.26	44.16	201,590.07	1,163.03	201,736.56	146.49
5	971.4818	51.47	201,358.33	998.2423	201,497.80	139.47
6	856.3243	58.39	201,254.73	880.1603	201,389.81	135.08
7*	**769.4031**	**64.99**	**201,226.23**	**790.9983**	**201,358.50**	**132.28**
8	701.2609	71.30	201,245.08	721.077	201,375.58	130.50
9	646.2796	77.37	201,295.34	664.6448	201,424.78	129.43
10	600.8994	83.21	201,367.23	618.056	201,496.09	128.87
11	562.7494	88.85	201,454.40	578.8818	201,583.06	128.66
12	530.1867	94.31	201,552.60	545.4389	201,681.33	128.73
13	502.0355	99.59	201,658.85	516.5219	201,787.87	129.02
14	477.4311	104.73	201,771.04	491.2447	201,900.50	129.46
15	455.7232	109.72	201,887.62	468.9401	202,017.65	130.04
The Buyer's Independent decision						
n	**Q*(1)**	**No of Shipments**	**ETC_b(Q*)**	**ETC_v(Q*)**	**ETC(Q*) Independent**	**ETC(n, Q*(n)) Integrated**
1	1581.3539	31.62	12073.92	196385.54	**208459.45**	**201,226.23**

Figure 4.3



The comparative chart in table 4.1 shows that the minimum Expected Total Cost is lower and this model gives better result than (Hsu & Hsu, 2012) where screening process had done at buyer site. An optimal result is indicated by bold letters.

4.8.2 Sensitivity Analysis with respect to F (freight cost)

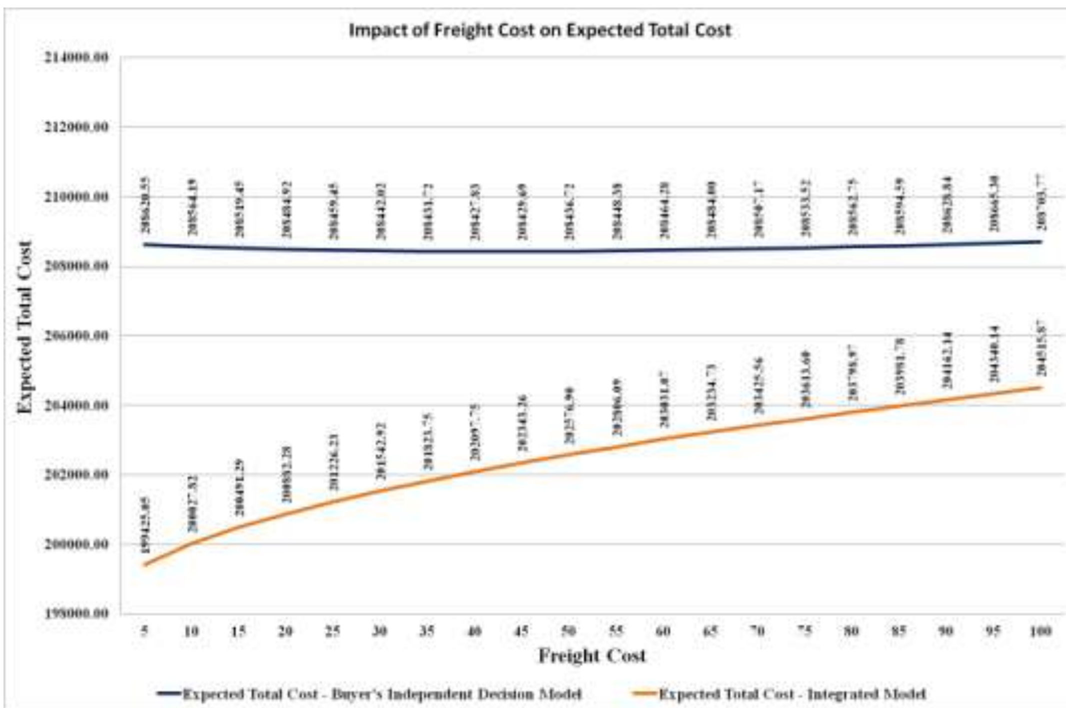
The minimum Expected Total Cost for the integrated solution with respect to different F where P=160,000, D=50,000, S_v=300, K=100, h_v=2, h_b=5, c_i=.0.5, c_w=50, c_r=100, $c_{\alpha\beta}$=200, c_{av}=300, $\beta = \lambda = \eta = 0.04$

Table 4.2

F	Buyer's independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*)	ETC_v (Q_b*)	ETC (Q_b*)	n*	Q*(n*)	No of Shipments to meet demand D	ETC(n*,Q *(n*))	
5	1449.33	11413.64	197206.91	208620.55	15	356.78	140.14	199425.05	9195.49
10	1483.44	11584.22	196979.97	208564.19	11	489.16	102.22	200027.82	8536.36
15	1516.78	11750.96	196768.48	208519.45	9	597.94	83.62	200491.29	8028.16
20	1549.40	11914.12	196570.80	208484.92	8	677.48	73.80	200882.28	7602.64
25	**1581.35**	**12073.92**	**196385.53**	**208459.45**	**7**	**769.40**	**64.99**	**201226.23**	**7233.23**

F	Buyer's independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*)	ETC_v (Q_b*)	ETC (Q_b*)	n*	Q*(n*)	No of Shipments to meet demand D	ETC(n*,Q*(n*))	
30	1612.67	12230.55	196211.47	208442.02	6	879.37	56.86	201542.92	6899.10
35	1643.39	12384.19	196047.53	208431.72	6	901.82	55.44	201823.75	6607.97
40	1673.55	12535.02	195892.81	208427.83	6	923.73	54.13	202097.75	6330.08
45	1703.17	12683.17	195746.52	208429.69	5	1059.97	47.17	202343.26	6086.43
50	1732.29	12828.79	195607.92	208436.72	5	1080.97	46.25	202576.90	5859.82
55	1760.92	12972.01	195476.38	208448.38	5	1101.56	45.39	202806.09	5642.29
60	1789.10	13112.93	195351.36	208464.28	5	1121.77	44.57	203031.07	5433.21
65	1816.84	13251.66	195232.33	208484.00	4	1300.87	38.44	203234.73	5249.27
70	1844.16	13388.31	195118.86	208507.17	4	1320.43	37.87	203425.56	5081.61
75	1871.08	13522.97	195010.55	208533.52	4	1339.71	37.32	203613.60	4919.92
80	1897.62	13655.71	194907.03	208562.75	4	1358.71	36.80	203798.97	4763.78
85	1923.80	13786.63	194807.97	208594.59	4	1377.45	36.30	203981.78	4612.81
90	1949.62	13915.78	194713.06	208628.84	4	1395.94	35.82	204162.14	4466.70
95	1975.11	14043.25	194622.05	208665.30	4	1414.19	35.36	204340.14	4325.16
100	2000.27	14169.10	194534.67	208703.77	4	1432.21	34.91	204515.87	4187.89

Figure 4.4



The table 4.2 shows Expected Total Cost for buyer's independent and integrated solution for different freight costs. It is observed as freight cost increased the number of lots per production batch from the vendor to the buyer is decreased and size of the lots is increased. The cost reduction of an integrated model of buyer's independent decision is higher for smaller freight cost and it decreased as freight cost increased. The freight cost value that is considered for calculation of optimal value in table 4.1 is shown in table 4.2 in bold letters.

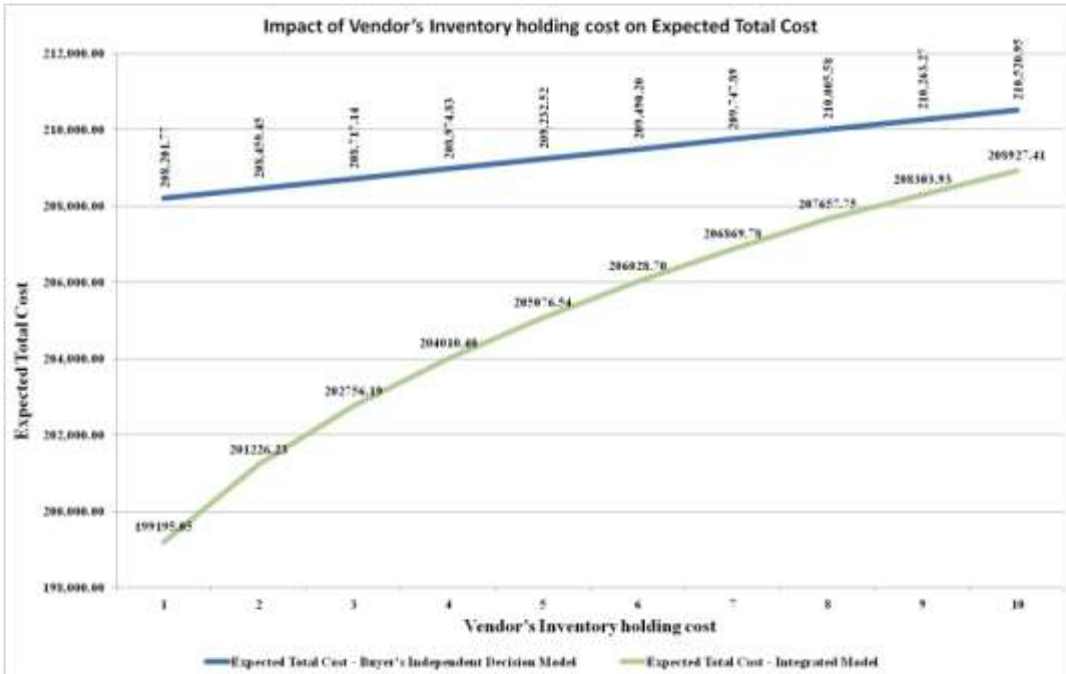
4.8.3 Sensitivity Analysis with respect to h_v (Vendor's Inventory holding cost)

The minimum Expected Total Cost for the integrated solution with respect to different h_v where P=160,000, D=50,000, S_v=300, K=100, h_b=5, F=25, c_i=.0.5, c_w=50, c_r=100, $c_{\alpha\beta}$=200, c_{av}=300, $\beta = \lambda = \eta = 0.04$

Table 4.3

h_v	Buyer's independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	$ETC_b(Q_b*)$	$ETC_v(Q_b*)$	$ETC(Q_b*)$	n*	Q*(n*)	No of Shipments to meet demand D	ETC(n*,Q*(n*))	
1	1,581.35	12,073.92	196,127.84	208,201.77	10	751.81	66.51	199,195.05	9,006.72
2	**1,581.35**	**12,073.92**	**196,385.53**	**208,459.45**	**7**	**769.40**	**64.99**	**201,226.23**	**7,233.23**
3	1,581.35	12,073.92	196,643.22	208,717.14	6	751.03	66.58	202,756.19	5,960.95
4	1,581.35	12,073.92	196,900.91	208,974.83	5	780.14	64.09	204,010.40	4,964.42
5	1,581.35	12,073.92	197,158.59	209,232.52	4	860.59	58.10	205,076.54	4,155.97
6	1,581.35	12,073.92	197,416.28	209,490.20	3	1,023.05	48.87	206,028.70	3,461.50
7	1,581.35	12,073.92	197,673.97	209,747.89	3	970.34	51.53	206,869.78	2,878.11
8	1,581.35	12,073.92	197,931.66	210,005.58	2	1,315.40	38.01	207,657.75	2,347.83
9	1,581.35	12,073.92	198,189.34	210,263.27	2	1,267.54	39.45	208,303.93	1,959.34
10	1,581.35	12,073.92	198,447.03	210,520.95	2	1,224.55	40.83	208,927.41	1,593.54

Figure 4.5



The table 4.3 shows Expected Total Cost for buyer's independent and integrated solution for different vendor's holding cost. It is observed as vendor's inventory holding cost increased the number of lots per production batch from the vendor to the buyer is decreased and size of lots is also increased. The cost reduction of integrated model from buyer's independent decision is higher for smaller vendor's holding cost and it decreased as vendor's holding cost increased. The vendor's inventory holding cost value that is considered for calculation of optimal value in table 4.1 is shown in table 4.3 in bold letters.

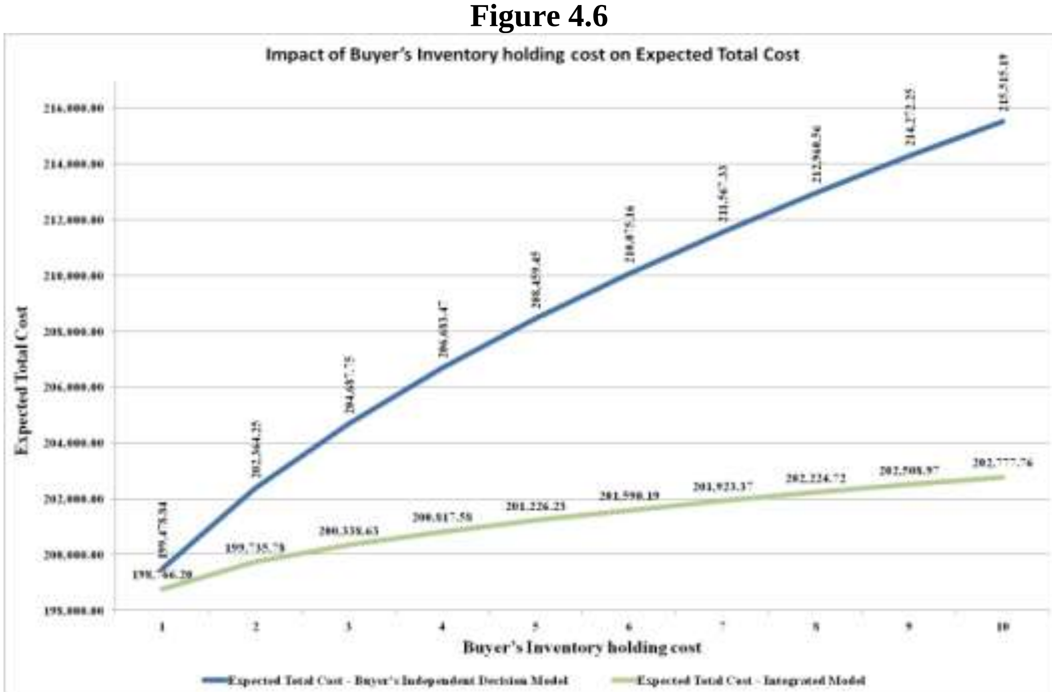
4.8.4 Sensitivity Analysis with respect to h_b (Buyer's Inventory holding cost)

The minimum Expected Total Cost for the integrated solution with respect to different h_b where P=160,000, D=50,000, S_v=300, K=100, h_v=2, F=25, c_i=.0.5, c_w=50, c_r=100, $c_{\alpha\beta}$=200, c_{av}=300, $\beta = \lambda = \eta = 0.04$

Table 4.4

h_b	Buyer's independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*)	ETC_v (Q_b*)	ETC(Q_b*)	n*	Q*(n*)	No of Shipments to meet demand D	ETC(n*,Q*(n*))	
1	3,536.01	7,701.94	19,1776.91	199,478.84	2	2,738.17	18.26	198,766.20	712.65
2	2,500.34	9,167.01	19,3197.23	202,364.25	4	1,360.71	36.75	199,735.78	2,628.47
3	2,041.52	10,291.21	19,4396.55	204,687.75	5	1,072.64	46.61	200,338.63	4,349.12
4	1,768.01	11,238.94	19,5444.53	206,683.47	6	892.77	56.01	200,817.58	5,865.89
5	**1,581.35**	**12,073.92**	**19,6385.53**	**208,459.45**	**7**	**769.40**	**64.99**	**201,226.23**	**7,233.23**
6	1,443.57	12,828.79	19,7246.36	210,075.16	8	679.35	73.60	201,590.19	8,484.96
7	1,336.49	13,522.97	19,8044.36	211,567.33	9	610.61	81.89	201,923.37	9,643.96
8	1,250.17	14,169.10	19,8791.47	212,960.56	9	594.85	84.05	202,224.72	10,735.84
9	1,178.67	14,775.95	19,9496.30	214,272.25	10	543.55	91.99	202,508.97	11,763.28
10	1,118.19	15,349.93	20,0165.27	215,515.19	10	531.61	94.05	202,777.76	12,737.42

Figure 4.6



The table 4.4 shows Expected Total Cost for the buyer's independent and integrated solution for different buyer's holding cost. It is observed that the impact of buyer's holding cost is just opposite to the vendor's holding cost. As the buyer's holding cost increased, the number of lots per production batch from the vendor to the buyer, is also increasing and size of lots is decreasing. The cost reduction of integrated model from buyer's independent decision is lesser for smaller buyer's holding cost and it increases as the buyer's holding cost is increasing. The Buyer's Inventory holding cost value that is considered for calculation of optimal value in table 4.1 is shown in table 4.4 in bold letters.

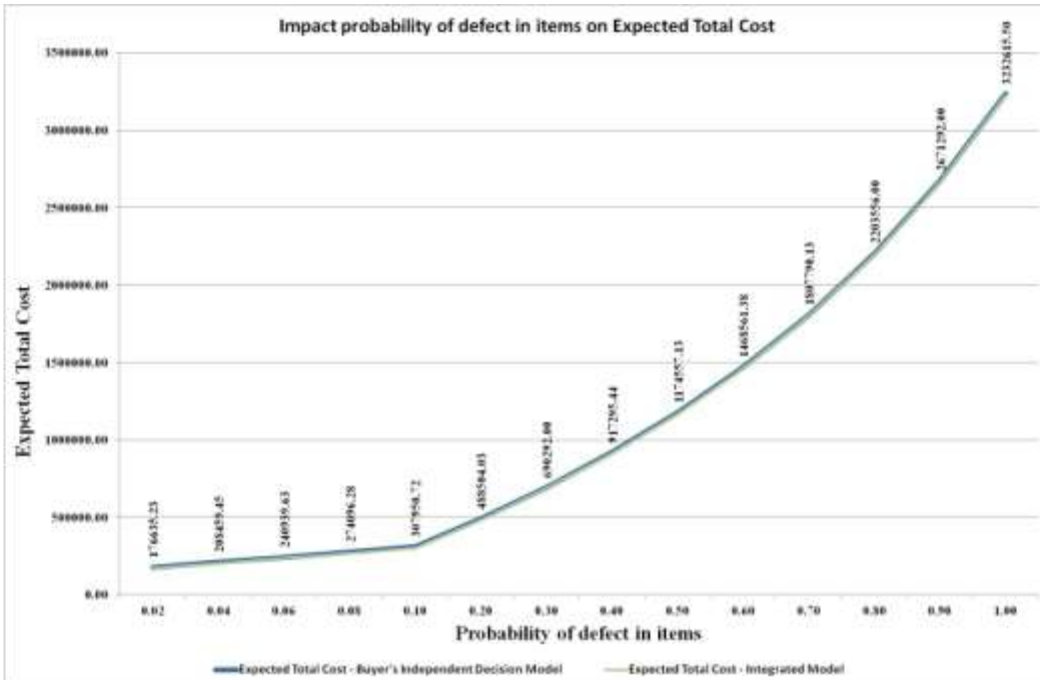
4.8.5 Sensitivity Analysis with respect to probability of defects

The minimum Expected Total Cost for the integrated solution for probability of defects when probability of defect percentage is uniformly distributed between 0 and β where P=160,000, D=50,000, S_v=300, K=100, h_v=2, h_b=5, F=25, c_i=.0.5, c_w=50, c_r=100, $c_{\alpha\beta}$=200, c_{av}=300, $\lambda = \eta = 0.04$

Table 4.5

β	Buyer's independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	$ETC_b(Q_b*)$	$ETC_v(Q_b*)$	$ETC(Q_b*)$	n*	Q*(n*)	No of Shipments to meet demand D	ETC(n*,Q*(n*))	
0.02	1,581.27	9,968.75	166,666.48	176,635.23	7	769.41	64.98	169,408.46	7,226.77
0.04	**1,581.35**	**12,073.92**	**196,385.53**	**208,459.45**	**7**	**769.40**	**64.99**	**201,226.23**	**7,233.23**
0.06	1,581.49	14,222.49	226,717.14	240,939.63	7	769.38	64.99	233,700.26	7,239.36
0.08	1,581.69	16,415.81	257,680.45	274,096.28	7	769.34	64.99	266,851.08	7,245.20
0.1	1,581.95	18,655.32	289,295.41	307,950.72	7	769.28	65.00	300,700.06	7,250.66
0.2	1,584.36	30,599.35	457,904.69	488,504.03	7	768.67	65.05	481,232.17	7,271.86
0.3	1,589.06	43,948.62	646,343.38	690,292.00	7	767.39	65.16	683,011.69	7,280.31
0.4	1,596.85	58,966.73	858,328.69	917,295.44	7	765.26	65.34	910,023.63	7,271.81
0.5	1,608.78	75,987.72	1,098,569.38	1,174,557.13	7	762.04	65.61	1167,315.54	7,241.59
0.6	1,626.34	95,441.26	1,373,120.13	1,468,561.38	7	757.48	66.01	1,461,376.44	7,184.94
0.7	1,651.57	117,889.76	1,689,900.38	1,807,790.13	6	838.93	59.60	1,800,685.32	7,104.81
0.8	1,687.31	144,083.78	2,059,472.13	2,203,556.00	6	831.14	60.16	2,196,552.45	7,003.55
0.9	1,737.64	175,048.13	2,496,243.75	2,671,292.00	6	821.33	60.88	2,664,406.97	6,885.03
1.0	1,808.37	212,219.52	3,020,396.00	3,232,615.50	5	928.58	53.85	3,225,821.88	6,793.62

Figure 4.7



The table 4.7 shows Expected Total Cost for the buyer's independent and integrated solution for different defective percentage β where defective percentage is uniformly distributed between 0 and β. As β increases the cost reduction decreased and Expected Total Cost in both situations increased. The Buyer's Inventory holding cost value is considered for calculation of optimal value in table 4.1 is shown in table 4.5 in bold letters.

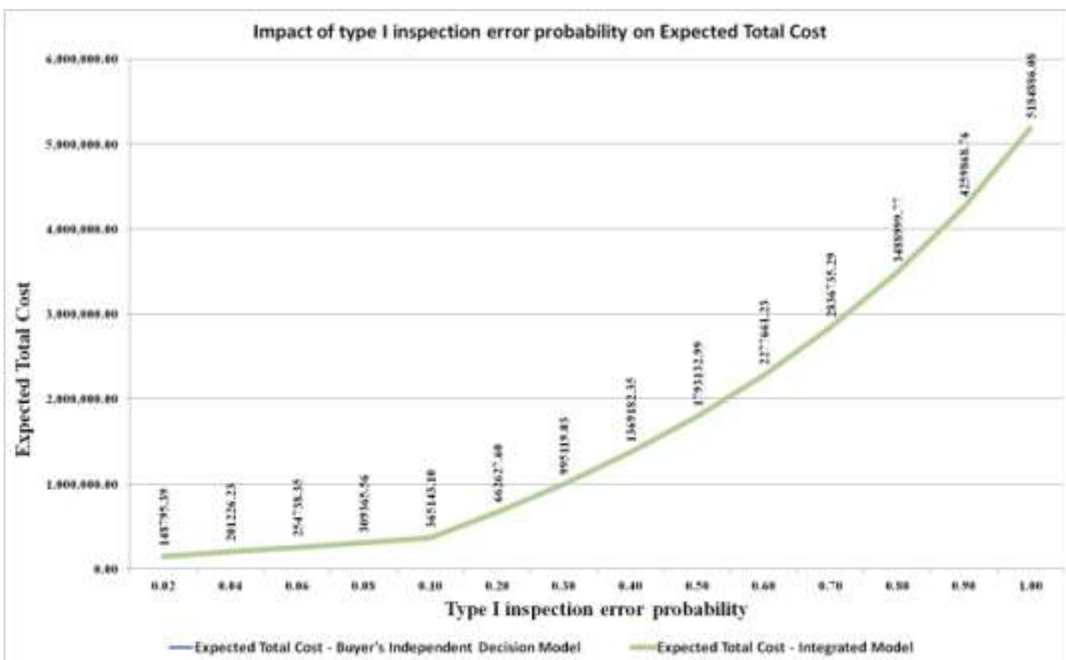
4.8.6 Sensitivity Analysis with respect to probability of type I inspection error percentage e_1

The minimum Expected Total Cost for the integrated solution for different values of type I inspection error percentage e_1 which is uniformly distributed between 0 and λ and where P=160,000, D=50,000, S_v=300, K=100, h_v=2, h_b=5, F=25, c_i=.0.5, c_w=50, c_r=100, $c_{\alpha\beta}$=200, c_{av}=300, $\beta = \eta = 0.04$

Table 4.6

λ	Buyer's independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*)	ETC_v (Q_b*)	ETC (Q_b*)	n*	Q*(n*)	No of Shipments to meet demand D	ETC (n*,Q*(n*))	
0.02	1,581.27	12,031.81	143,992.72	156,024.53	7	769.48	64.98	148,795.39	7,229.14
0.04	**1,581.35**	**12,073.92**	**196,385.53**	**208,459.45**	**7**	**769.40**	**64.99**	**201,226.23**	**7,233.23**
0.06	1,581.50	12,116.89	249,858.41	261,975.30	7	769.31	64.99	254,738.35	7,236.94
0.08	1,581.70	12,160.76	304,445.06	316,605.81	7	769.20	65.00	309,365.56	7,240.25
0.10	1,581.97	12,205.55	360,180.72	372,386.25	7	769.06	65.01	365,143.10	7,243.15
0.20	1,584.49	12,444.44	657,433.50	669,877.94	7	768.02	65.10	662,627.60	7,250.34
0.30	1,589.42	12,711.52	989,649.94	1,002,361.44	7	766.25	65.25	995,119.03	7,242.41
0.40	1,597.61	13,012.19	1,363,384.88	1,376,397.13	7	763.53	65.49	1369,182.35	7,214.78
0.50	1,610.23	13,353.49	1,786,941.63	1,800,295.13	7	759.62	65.82	1,793,132.99	7,162.13
0.60	1,628.87	13,744.71	2,270,995.50	2,284,740.25	7	754.23	66.29	2,277,661.23	7,079.02
0.70	1,655.80	14,198.51	2,829,510.50	2,843,709.00	6	834.61	59.91	2,836,735.29	6,973.71
0.80	1,694.18	14,732.66	3,481,106.75	3,495,839.50	6	825.66	60.56	3,488,999.77	6,839.73
0.90	1,748.55	15,372.81	4,251,182.50	4,266,555.50	6	814.52	61.39	4,259,868.76	6,686.74
1.00	1,825.49	16,157.24	5,175,307.00	5,191,464.00	5	920.16	54.34	5,184,886.08	6,577.92

Figure 4.8



The table 4.8 shows Expected Total Cost for buyer's independent and integrated solution for different values of type I inspection error percentage e_1 which is uniformly distributed between 0 and λ. As λ increases the cost reduction decreased and Expected Total Cost in both situations increased rapidly. The Buyer's Inventory holding cost value that is considered for calculation of optimal value in table 4.1 is shown in table 4.6 in bold letters.

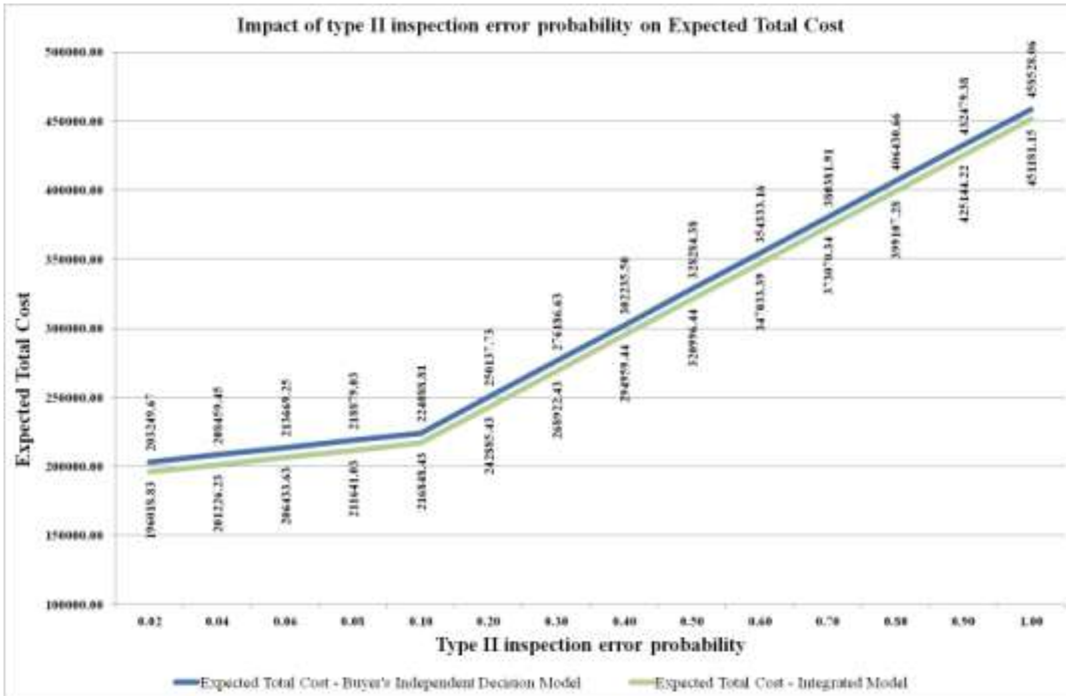
4.8.7 Sensitivity Analysis with respect to probability of type II inspection error e_2

The minimum Expected Total Cost for the integrated solution for probability of type II inspection error e_2 which is uniformly distributed between 0 and η and where P=160,000, D=50,000, S_v=300, K=100, h_v=2, h_b=5, F=25, c_i=.0.5, c_w=50, c_r=100, $c_{\alpha\beta}$=200, c_{av}=300, $\beta = \lambda = 0.04$

Table 4.7

η	Buyer's independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*)	ETC_v (Q_b*)	ETC (Q_b*)	n*	Q*(n*)	No of Shipments to meet demand D	ETC (n*,Q*(n*))	
0.02	1,581.36	9,989.81	193,259.86	203,249.67	7	769.33	64.99	196,018.83	7,230.85
0.04	**1,581.35**	**12,073.92**	**196,385.53**	**208,459.45**	**7**	**769.40**	64.99	**201,226.23**	**7,233.23**
0.06	1,581.35	14,158.03	199,511.22	213,669.25	7	769.47	64.98	206,433.63	7,235.62
0.08	1,581.35	16,242.14	202,636.89	218,879.03	7	769.55	64.97	211,641.03	7,238.00
0.10	1,581.35	18,326.25	205,762.56	224,088.81	7	769.62	64.97	216,848.43	7,240.38
0.20	1,581.34	28,746.79	221,390.94	250,137.73	7	769.97	64.94	242,885.43	7,252.30
0.30	1,581.34	39,167.34	237,019.28	276,186.63	7	770.33	64.91	268,922.43	7,264.19
0.40	1,581.34	49,587.87	252,647.63	302,235.50	7	770.68	64.88	294,959.44	7,276.06
0.50	1,581.34	60,008.40	268,275.97	328,284.38	7	771.04	64.85	320,996.44	7,287.94
0.60	1,581.34	70,428.90	283,904.25	354,333.16	7	771.39	64.82	347,033.39	7,299.77
0.70	1,581.35	80,849.40	299,532.50	380,381.91	7	771.75	64.79	373,070.34	7,311.57
0.80	1,581.36	91,269.88	315,160.78	406,430.66	7	772.10	64.76	399,107.28	7,323.38
0.90	1,581.38	101,690.36	330,789.00	432,479.38	7	772.46	64.73	425,144.22	7,335.16
1.00	1,581.39	112,110.83	346,417.22	458,528.06	7	772.81	64.70	451,181.15	7,346.92

Figure 4.9



The table 4.9 shows Expected Total Cost for buyer's independent and integrated solution for different values of type II inspection error percentage e_2 which is uniformly distributed between 0 and η. As η increases Expected Total Cost in both situations increased. The type II inspection error e_2 value that is considered for calculation of optimal value in table 4.1 is shown in table 4.7 in bold letters.

4.9 Conclusions of the model

This research work is about, an integrated single-vendor single buyer inventory model with imperfect production quality and imperfect inspection error. Previous research focused on conducting inspection process at the buyer site after receiving fresh lot. In this paper inspection is being performed particularly at the vendor's site. Our objective is to find minimum total joint costs incurred for the model. It is assumed that production process is not perfect and produces some defective items with a known probability density function. After production, all items go through an inspection process. The inspection process is done parallel to production of items. An item is available for inspection just after its production. The inspection process classifies items into non-defective and defective items. Defective Items are separated, stored in inventory and disposed at end of each production cycle. Non-defective Items are sent to the buyer in equal size lots Q to meet market demands. The inspection process is also not perfect. There is a chance that the inspection process may be classify a non-defective item as defective (type I inspection error) or defective items as non-defective (type II inspection error). The expected total annual cast for the vendor and the buyer are derived. For integrated vendor and buyer procedure is provided to find out optimal minimum annual cost. The minimum cast of this model is compared with a model where inspection process is conducted at the buyer site just after receiving fresh lot. Numerical example shows a significant reduction in expected total cost when inspection process is done at the vendor site. We observe that there is reduction of lot size that has been shipped to the buyer.

We found that other related research works, deals with inspection of items at buyer site, performed just after receiving a new fresh lot of items.

CHAPTER 5

An integrated single-vendor, single-buyer inventory model for imperfect quality production, imperfect inspection at vendor site and with backorder

5.1 Introduction

(Salameh & Jaber, 2000) had pointed out that items received by the buyer in traditional production/inventory models were not of perfect quality. They had extended traditional economical order quantity (EOQ) model by taking into account of defective items received by a vendor with a known probability density function. After receiving a lot, the vendor conducted 100% screening process at a rate of x unit per unit time where $x > D$ (the demand rate) before selling them in market. During the screening process, Items classified as defective were kept in inventory and sold-out at discounted price at the end of each lot cycle. (Wee et al., 2007) extended (Salameh & Jaber, 2000) EOQ model and allowed shortage backordering by adding an assumption that customers were willing to wait till the next supply to arrive, whenever there was shortage of items. (Maddah & Jaber, 2008) applied renewal and reward theorem to rectify a flaw in (Salameh & Jaber, 2000) EOQ model to get an exact expression for expected profit. (Khan et al., 2011) stated that the inspection process was itself imperfect. Items could be classified wrongly. For example, defective items could be classified as non-defective and non-defective as defective. Defective items sold (due to inspection error) in the market were returned back by the customers for replacement with fresh items and in turn stored in the inventory. At the end of each screening process, all defective items returned back by the customers along with items identified defective during the screening process were returned back to the vendor for disposal. Classifying non-defective items as defective also caused loss of profit because non-defective items were sold at discounted price. Because of screening errors, inventory level increased and extra demand of items was created. (Hsu & Hsu, 2012), through a technical note corrected an assumption of (Wee et al., 2007) that all, backorder were cleared immediately as soon as a fresh lot of the items arrived, ignoring the time required for screening of items.

Above research works assumed that screening of items was done at buyer site which was first proposed by (Salameh & Jaber, 2000). Works that are done after (Salameh & Jaber, 2000) further extended this assumption. In this paper, it is assumed that screening is done at vendor site, along with production of items, before a lot of items are sent to the buyer".

5.2 Notation and Assumptions

Flowing notations and assumptions are used

Q_P : the count of items of a lot that are produced per production cycle

Q : the count of items of a lot that delivered from vendor to buyer

n : the number of deliveries to buyer per production cycle, a positive integer $(Q_P = nQ)$

D : the demand rate (demand per year)

P : the production rate of items ($P > D$) at vendor

x : the Items screening/inspection rate of items

S_v : the Setup cost per production cycle

K : the ordering cost per order for the buyer

C_i : the vendor's inspection cost per unit

C_w : the vendor's cost per unit for producing defective items (warranty cost)

$C_{\alpha\beta}$: the buyer's cost per unit for selling defective items in the market (due to Type II Error)

C_{av} : the vendor's cost of selling defective items in the market (due to Type II Error)

C_r : the vendor's cost for rejecting non-defective items as defective items

h_v : the inventory holding cost per unit item of vendors

h_b : the inventory holding cost per unit item for buyer

F : the transportation cost per delivery of items

b : Backordering cost per unit per unit time

p : the probability of production of defective items

$f(p)$: the probability density function of p

e_1 : the probability of type I screening Error (Classifying Non-Defective item as Defective items)

$f(e_1)$: the probability density function of e_1

e_2 : the probability of type II screening Error (Classifying Defective item as Non-Defective items)

$f(e_2)$: the probability density function of e_2

B_1 : the number of items classified as defective after screening per production lot

B_2 : the number of defective items classified as Non-Defective item per production lot (Type II Screening Error)

B_3 : the number of backorder shortage items allowed

T : the time interval between two successive deliveries to buyers of Q items

T_1 : the time period during which vendor produce items

T_2 : the time period during which vendor supplies items from inventory

T_C : the production cycle time ($T_C = nT$)

***** : the superscript to represent optimal value

In manufacturing industries, the vendor produce items at production rate of P greater than the demand rate D of the items. Item produced contains some defective items with probability *p* and probability density function *f (p)*. To improve quality of items supplied to consumer, Inspectors, at the vendor site, conduct 100% screening test of items produced. The inspection of all items is performed to filter out B_1 defective items. To save time, the inspection is done along with production. The inspection rate x is greater than the production rate P of items (x > P) so that items that produced will be inspected without any delay in manufacturing industries. Inspectors are human. Due to human error, the screening test is assumed imperfect; there is a possibility that inspectors could do mistakes in classifying items. Items produced in one production setup cycle are sent to the buyer in n multiple lots after their inspection. It is assumed that for reduction in total expected cost of inventories, the buyer and the vendor agree to have some shortage (backorder) in the buyer's inventory with the condition that if reduction of inventory carrying cost is more than loss occurred due to shortage of items.

There are two types of screening error that inspectors may commit.

Type I : Inspector incorrectly classifies non-defective items as defective items with probability e_1 and probability density function $f(e_1)$

Type II : Inspector incorrectly classifies defective items as non-defective items with probability e_2 and probability density function $f(e_2)$

5.3 Mathematical Model

Figure 5.1

Vendor's Inventory

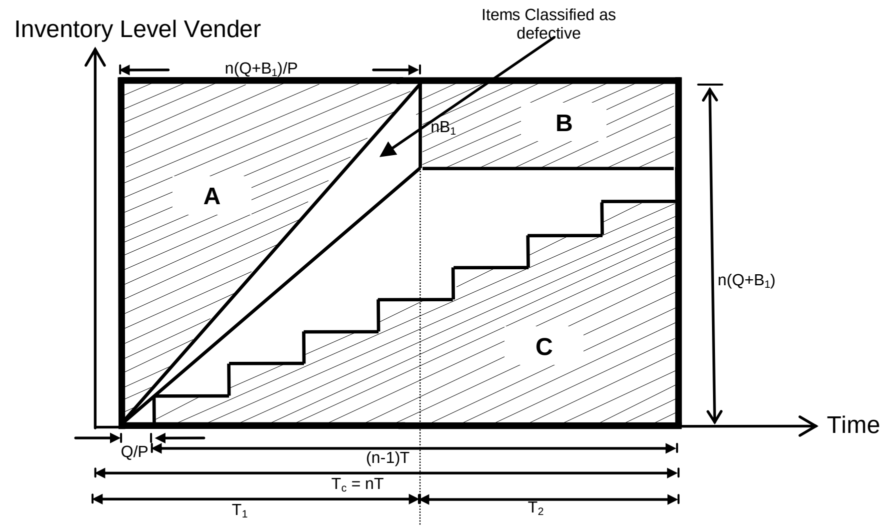
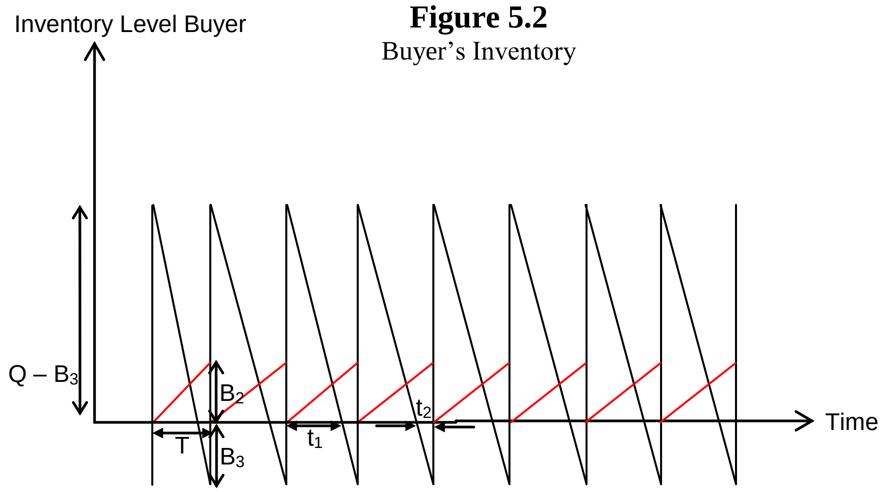


Figure 5.2

Buyer's Inventory



1. Number of defective items produced in a production lot $\mathbf{Q + B_1}$ with its probability distribution **p** is

$$= (Q + B_1)p \tag{5.1}$$

2. Number of non-defective items produced in a production lot $\mathbf{Q + B_1}$ with probability distribution **p** to produce defective items is

$$= (Q + B_1)(1 - p) \tag{5.2}$$

3. Number of items classified as defective which are actually non-defective due to Type I screening Error $\mathbf{e_1}$ in a production lot $\mathbf{Q + B_1}$, probability distribution $\mathbf{p}$ to produce defective items in production is

(Non-Defective → Defective)

$$= (Q + B_1)(1 - p)e_1 \tag{5.3}$$

4. Number of items classified as non-defective which are actually defective due to Type II screening Error $\mathbf{e_2}$ in a production lot $\mathbf{Q + B_1}$, probability distribution $\mathbf{p}$ to produce defective items in production is

(Defective → Non-Defective)

$$= (Q + B_1)pe_2 \tag{5.4}$$

5. Number of items classified as defective which are actually defective, considering Type I screening Error $\mathbf{e_1}$ and Type II screening Error $\mathbf{e_2}$ in a production lot $\mathbf{Q + B_1}$, probability distribution $\mathbf{p}$ to produce defective items in production is

(Defective → Defective)

$$= (Q + B_1)p(1-e_2) \tag{5.5}$$

6. Number of items classified as non-defective which are actually non-defective, considering Type I screening Error $\mathbf{e_1}$ and Type II screening Error $\mathbf{e_2}$ in a production lot $\mathbf{Q + B_1}$, probability distribution $\mathbf{p}$ to produce defective items in production is

(Non-Defective → Non-Defective)

$$= (Q + B_1)\{(1-p)(1-e_1)\} \tag{5.6}$$

B_1 is the total items which are classified as defective after the screening process. It includes defective items classified as defective (Eq – 5) and non-defective items classified defective (Eq – 3). Thus

$$B1 = (Q + B_1)\ \{(1 - p)e_1 + p(1 - e_2)\} \tag{5.7}$$

B_2 is those items which are classified as non-defective but are actually defective. From Eq-4 we get

$$B2 = (Q + B1)pe_2 \tag{5.8}$$

B_1 is number of items that are being classified as defective items, due to type I screening error. B_1items are produced additional to the Q items by the vendor to enable him to supply Q items to buyer at the beginning of each replenishment cycle. Thus, for each supply of Q items the vendor produced $(Q + B_1)$ items.

Thus B_1= Defective items classified as defective + Type I error (classify non-defective as defective)

$B_1 = (Q + B_1)p(1 - e_2) + (Q + B_1)(1 - p)e_1$

after solving it,

$$B_1 = \frac{p(1\text{-}e_2) + (1\text{-}p)e_1}{1\text{-}\{p(1\text{-}e_2) + (1\text{-}p)e_1\}} Q \tag{5.9}$$

B_2 items are defective items due to type II Error. (Classify defective as non-defective)

$B_2 = (Q + B_1)pe_2$

Putting the value of B_1

$$B_2 = \frac{pe_2 Q}{1\text{-}\{p(1\text{-}e_2) + (1\text{-}p)e_1\}} \tag{5.10}$$

Where Type I Error = $p(1\text{-}e_2)+(1\text{-}p)e_1$

Type II Error = pe_2

Defective items, $B_2$are sold in the market and replaced by consumers with fresh items. Replacements of defective items create additional demand of fresh items. The effective demand D' includes both real market demand *D* and replacement demands$\left(\frac{B_2}{T}\right)$.Thus, effective demand$D' = D + \frac{B_2}{T}$.

The cycle length of each delivery *T* for the buyer is $T = \frac{Q}{D'}$. Substituting value of D'

$T = \frac{Q}{D+\frac{B_2}{T}} = \frac{QT}{DT+B_2}$ $i.e.,\ DT + B_2 = Q\ \ i.e., T = \frac{Q-B_2}{D}$

Solving above,

$$T = \frac{Q - B_2}{D}$$

$$\mathrm{T} = \left[\mathrm{Q} - \frac{\mathrm{pe_2Q}}{1\text{-}\{\mathrm{p}(1\text{-}\mathrm{e_2})+(1\text{-}\mathrm{p})\mathrm{e_1}\}}\right]\frac{1}{\mathrm{D}}$$

$$\mathrm{T} = \frac{(1-\mathrm{p})(1-\mathrm{e_1})\mathrm{Q}}{[1-\{\mathrm{p}(1-\mathrm{e_2})+(1-\mathrm{p})\mathrm{e_1}\}]\mathrm{D}} \tag{5.11}$$

5.4 The Buyer's cost per production cycle

The buyer gets Q items at the beginning of each replenishment cycle. B_3 items are immediately sold out to meet backorder requirement and Q - B_3 items are stored in buyer's inventory (refer figure 5.2). Defective items B_2(replaced by customers and stored in buyer's inventory) are sent to vendor at the end of each cycle. The vendor disposes these defective items immediately after receiving from the buyer. The inventory holding cost of the buyer is

HC_b = Holding Cost of Q - B_3 items for t_1 time + Holding Cost of B_2 items for T time

$$HC_b = h_b \frac{(Q - B_3)}{2} t_1 + h_b \frac{B_2}{2} T$$

Here $t_1 = \frac{Q - B_3}{D}$ (5.12)

Substituting values of B_2 (Equation - 5.2), T (Equation – 5.3) and t_1 (Equation – 5.4) and solving, holding cost to the buyer is

$$HC_b = h_b \frac{(Q-B_3)}{2} \frac{(Q-B_3)}{D} + h_b \frac{pe_2 Q}{1\text{-}\{p(1\text{-}e_2)+(1\text{-}p)e_1\}} \frac{(1\text{-}p)(1\text{-}e_1)Q}{1\text{-}\{p(1\text{-}e_2)+(1\text{-}p)e_1\}D}$$

$$HC_b = \frac{h_b}{2D}\left\{(Q - B_3)^2 + \frac{(1-p)(1-e_1)pe_2 Q^2}{[1-\{(1-e_2)+(1-p)e_1\}]^2}\right\}$$

Average backorder cost of B3 items for t2 time = $\frac{1}{2} bB_3(t_2) = \frac{bB_3^2}{2D}$ where $t_2 = \frac{B_2}{D}$

The total cost to the buyer per production cycle includes ordering cost, transportation cost, post-sale failure cost (due to sales of defective items), backordering cost and holding cost. These costs will depend on three variable parameters i.e., n (number of orders per production cycle), Q (lot size) and B_3 (number of items allowed to backorder). Thus, total cost per production cycle is

$$TC_b(n, Q, B_3) = K + nF + nc_{\alpha\beta}(Q+B_1)pe_2 + \frac{nbB_3^2}{2D} + \frac{nh_b}{2D}\left\{(Q - B_3)^2 + \frac{(1-p)(1-e_1)pe_2 Q^2}{[1-\{(1-e_2)+(1-p)e_1\}]^2}\right\}$$

$$TC_b(n, Q, B_3) = K + nF + \frac{nc_{\alpha\beta} pe_2 Q}{1-\{p(1-e_2)+(1-p)e_1\}} + \frac{nbB_3^2}{2D} + \frac{nh_b}{2D}\left\{(Q - B_3)^2 + \frac{(1-p)(1-e_1)pe_2 Q^2}{[1-\{(1-e_2)+(1-p)e_1\}]^2}\right\} \quad (5.13)$$

5.5 The Vendor's cost per production cycle

At the vendor's site, screening test is conducted immediately after production of items to classify them as defective or non-defective. After screening, Q non-defective items are sent to the buyer inn lots at an equal interval of time *T*. Surplus items are kept at vendor's inventory for future supply.

Items, classified as defective, are also kept in inventory. When production finished all defective items are sold at discounted price. In figure nB_1 shows inventory of defective items

Figure 5.1 shows that vendor's holding cost per production cycle can be obtained as (see, for example, (Goyal et al., 2003), (Huang, 2004) and (Hsu & Hsu, 2012))

Inventory holding cost per cycle = h_v [bold area – shaded area A, B and C]

$$= h_v \left[n(Q+B_1)\left\{\frac{Q}{P} + (n\text{-}1)T\right\} - \frac{1}{2}n(Q+B_1)\frac{n(Q+B_1)}{P} - \left\{\frac{Q}{P} + (n\text{-}1)T - \frac{n(Q+B_1)}{p}\right\} nB_1 - \frac{n(n\text{-}1)TQ}{2}\right]$$

$$= \frac{h_v}{2P}\left[(2n - n^2)Q^2 + n(n\text{-}1)PTQ + n^2B_1^2\right]$$

Replacing the values of T and B_1

$$= \frac{h_v}{2P}\left[(2n - n^2)Q^2 + n(n\text{-}1)PQ\,\frac{(1\text{-}p)(1\text{-}e_1)Q}{[1\text{-}\{p(1\text{-}e_2)+(1\text{-}p)e_1\}]D} + n^2\frac{\{p(1\text{-}e_2)+(1\text{-}p)e_1\}^2Q^2}{[1\text{-}\{p(1\text{-}e_2)+(1\text{-}p)e_1\}]^2}\right]$$

$$= \frac{h_v}{2P}\left[(2n - n^2)Q^2 + \frac{n(n\text{-}1)(1\text{-}p)(1\text{-}e_1)PQ^2}{[1\text{-}\{p(1\text{-}e_2)+(1\text{-}p)e_1\}]D} + \frac{n^2\{p(1\text{-}e_2)+(1\text{-}p)e_1\}^2Q^2}{[1\text{-}\{p(1\text{-}e_2)+(1\text{-}p)e_1\}]^2}\right]$$

Let $A = 1 - \{p(1-e_2) + (1-p)e_1\}$ (5.14)

Then vendor's holding cost

$$= \frac{h_v}{2P}\left[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\right] \quad (5.15)$$

Adding the cost of setup, warranty, Type I errors, Type II errors and inventory holding, the vendor's total cost per production cycle TC_v (n, Q) is

Setup Cost = S_v

Warranty Cost = $n(Q + B_1)pC_w = \frac{npc_wQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{npc_wQ}{A}$

Type I Error = $n(Q + B_1)(1 - p)e_1C_r = \frac{n(1-p)e_1C_rQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{n(1-p)e_1C_rQ}{A}$

Type II Error = $n(Q + B_1)pe_2C_{av} = \frac{npe_2C_{av}Q}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{npe_2C_{av}Q}{A}$

Screening Cost = $n(Q + B_1)C_i = \frac{nC_iQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{nC_iQ}{A}$

$$TC_v(n,Q) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\Big[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\Big] \quad (5.16)$$

5.6 The integrated model for vendor-buyer

The total cost of the integrated model of the vendor-buyer per production cycle is

$$TC_C(n,Q,B_3) = TC_v(n,Q) + TC_b(n,Q,B_3)$$

$$TC_C(n,Q,B_3) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\Big[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\Big] + K + nF + \frac{nc_{\alpha\beta}pe_2\,Q}{A} + \frac{nbB_3^2}{2D} + \frac{nh_b}{2D}\Big\{(Q - B_3)^2 + \frac{(1-p)(1-e_1)pe_2Q^2}{A^2}\Big\}$$

$$TC_C(n,Q,B_3) = S_v + K + nF + \frac{nbB_3^2}{2D} + \frac{nh_b(Q-B_3)^2}{2D} + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{nc_{\alpha\beta}pe_2\,Q}{A} + \frac{h_v}{2P}\Big[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\Big] + \frac{nh_b(1-p)(1-e_1)pe_2Q^2}{2A^2D}$$

$$TC_C(n,Q,B_3) = S_v + K + nF + \frac{nbB_3^2}{2D} + \frac{nh_bB_3^2}{2D} + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} +$$

$$\frac{nC_iQ}{A} + \frac{nc_{\alpha\beta}pe_2\,Q}{A} - \frac{nh_bB_3Q}{D} +$$

$$\frac{h_v}{2P}\left[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\right] +$$

$$\frac{h_b(1-p)(1-e_1)pe_2Q^2}{2A^2D} + \frac{nh_bQ^2}{2D}$$

The expected total cost of the integrated model is

$$E[TC_C(n,Q,B_3)] = S_v + K + nF + \frac{n(b+h_b)B_3^2}{2D} + \frac{nc_wQE[p]}{E[A]} + \frac{nC_rQ(1-E[p])E[e_1]}{E[A]} +$$

$$\frac{nC_{av}QE[p]E[e_2]}{E[A]} + \frac{nC_iQ}{E[A]} + \frac{nc_{\alpha\beta}QE[p]E[e_2]}{E[A]} - \frac{nh_bB_3Q}{D} +$$

$$\frac{h_v}{2P}\left[(2n - n^2)Q^2 + \frac{n(n-1)PQ^2(1-E[p])(1-E[e_1])}{E[A]D} + \frac{n^2Q^2\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A^2]}\right] +$$

$$\frac{h_bQ^2}{2D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right] \tag{5.17}$$

Where

$$E[A] = 1 - \{E[p](1-E[e_2]) + (1-E[p])E[e_1]\} \tag{5.18}$$

Since the cycle time $T_c = \frac{n(1-p)(1-e_1)Q}{[1-\{p(1-e_2)+(1-p)e_1\}]D} = \frac{n(1-p)(1-e_1)Q}{AD}$,

Expected cycle time $E[T_C]$ is

$$E[T_c] = \frac{nQ(1-E[p])(1-E[e_1])}{DE[A]} \tag{5.19}$$

5.6.1 Application of Renewal and Reward theorem

Using the renewal and reward theorem, the expected total cost $ETC(n,Q,B_3)$ of the integrated model for vendor and buyer is

$$ETC(n,Q,B_3) = \frac{E[TC_c(n,Q,B_3)]}{E[T_c]}$$

$ETC(n, Q, B_3) =$

$$\left[S_v + K + nF + \frac{n(b+h_b)B_3^2}{2D}\right] * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])} + \frac{Q}{E[A]}\Big[nc_w E[p] + nC_r(1 - E[p])E[e_1] + nC_{av}E[p]E[e_2] + nC_i + nc_{\alpha\beta}E[p]E[e_2] - \frac{n\, h_b B_3 E[A]}{D}\Big] * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])} +$$

$$\frac{Q^2}{2}\left[\frac{h_v}{P}\left[\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right] + \frac{h_b}{D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right]\right] * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])}$$

$$ETC(n, Q, B_3) = \left[S_v + K + nF + \frac{n(b+h_b)B_3^2}{2D}\right] * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])} + \Big[nc_w E[p] + n(1 - C_r E[p])E[e_1] + nC_{av}E[p]E[e_2] + nC_i + nc_{\alpha\beta}E[p]E[e_2] - \frac{nh_b B_3 E[A]}{D}\Big] * \frac{D}{n(1-E[p])(1-E[e_1])} + \frac{DQE[A]}{2n(1-E[p])(1-E[e_1])}\left[\frac{h_v}{P}\left[\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right] + \frac{h_b}{D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right]\right] \tag{5.20}$$

5.6.2 Finding optimal solution

To estimate B_3 by differentiating of the above equation with respect to B_3

$$\frac{\partial ETC(n,Q,B_3)}{\partial B_3} = \frac{2n(b+h_b)B_3}{2D} \cdot \frac{DE[A]}{nQ(1-E[p](1-E[e_2])} - \frac{nE[A]h_b}{D} * \frac{D}{n(1-E[p])(1-E[e_1])}$$

$$\frac{\partial ETC(n,Q,B_3)}{\partial B_3} = \frac{n(b+h_b)B_3 E[A]}{nQ(1-E[p])(1-E[e_2])} - \frac{nh_b E[A]}{D} * \frac{D}{n(1-E[p])(1-E[e_1])} \tag{5.21}$$

Differentiating again with respect to B_3

$$\frac{\partial^2 ETC(n,Q,B_3)}{\partial {B_3}^2} = \frac{n(b+h_b)E[A]}{nQ(1-E[p])(1-E[e_2])} \tag{5.22}$$

Differentiating $\frac{\partial ETC(n,Q,B_3)}{\partial B_3} = \frac{n(b+h_b)B_3E[A]}{nQ(1-E[p])(1-E[e_2])} - \frac{nh_bE[A]}{D} * \frac{D}{n(1-E[p])(1-E[e_1])}$

with respect to Q

$$\frac{\partial ETC^2(n,Q,B_3)}{\partial B_3 \partial Q} = -\frac{n(b+h_b)B_3E[A]}{nQ^2(1-E[p])(1-E[e_2])}$$

Putting $\frac{\partial ETC(n,Q,B_3)}{\partial B_3} = 0$ to get optimal value of B_3

$$\frac{n(b+h_b)B_3E[A]}{nQ(1-E[p])(1-E[e_2])} - \frac{nh_bE[A]}{D} * \frac{D}{n(1-E[p])(1-E[e_1])} = 0$$

$$\frac{(b+h_b)B_3}{Q} = h_b$$

$$B_3 = \frac{h_b\, Q}{(b+h_b)}$$

Putting value of B_3 in $ETC(n,Q,B_3)$

$$ETC(n,Q) = [S_v + K + nF] * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])} + \frac{n(b+h_b)}{2D} * \left[\frac{h_b\, Q}{(b+h_b)}\right]^2 *$$

$$\frac{DE[A]}{nQ(1-E[p])(1-E[e_1])} + \Big[nc_wE[p] + nC_r(1-E[p])E[e_1] + nC_{av}E[p]E[e_2] +$$

$$nC_i + nc_{\alpha\beta}E[p]E[e_2]\Big] * \frac{D}{n(1-E[p])(1-E[e_1])} - \frac{n\, h_bE[A]}{D} * \frac{h_b\, Q}{(b+h_b)} * \frac{D}{n(1-E[p])(1-E[e_1])} +$$

$$\frac{DQE[A]}{2n(1-E[p])(1-E[e_1])}\Bigg[\frac{h_v}{P}\Bigg[\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} +$$

$$\frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\Bigg] + \frac{h_b}{D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right]\Bigg]$$

$$ETC(n,Q) = [S_v + K + nF] * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])} + \frac{{h_b}^2E[A]Q}{2(b+h_b)(1-E[p])(1-E[e_1])} +$$

$$\Big[nc_wE[p] + nC_r(1-E[p])E[e_1] + nC_{av}E[p]E[e_2] + nC_i + nc_{\alpha\beta}E[p]E[e_2]\Big] *$$

$$\frac{D}{n(1-E[p])(1-E[e_1])} - \frac{{h_b}^2E[A]Q}{(b+h_b)(1-E[p])(1-E[e_1])} + \frac{DQE[A]}{2n(1-E[p])(1-E[e_1])}\Bigg[\frac{h_v}{P}\Bigg[\left(2n - n^2\right) +$$

$$\frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2])+ (1-E[p])E[e_1]\}^2}{E[A^2]}\Bigg] +$$

$$\frac{h_b}{D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right]\Bigg]$$

$$ETC(n,Q) = [S_v + K + nF] * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])} + \Big[nc_wE[p] + nC_r(1 - E[p])E[e_1] +$$

$$nC_{av}E[p]E[e_2] \quad + nC_i + nc_{\alpha\beta}E[p]E[e_2]\Big] * \frac{D}{n(1-E[p])(1-E[e_1])} - \frac{{h_b}^2E[A]Q}{2(b+h_b)(1-E[p])(1-E[e_1])} +$$

$$\frac{DQE[A]}{2n(1-E[p])(1-E[e_1])}\Bigg[\frac{h_v}{P}\Big[\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} +$$

$$\frac{n^2\{E[p](1-E[e_2])+ (1-E[p])E[e_1]\}^2}{E[A^2]}\Big] + \frac{h_b}{D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right]\Bigg] \quad (5.23)$$

To estimate Q by differentiating of the above equation with respect to Q

$$\frac{\partial ETC(n,fQ)}{\partial Q} = -[S_v + K + nF] * \frac{E[A]D}{nQ^2(1-E[p])(1-E[e_1])} - \frac{{h_b}^2E[A]}{2(b+h_b)(1-E[p])(1-E[e_1])} +$$

$$\frac{DE[A]}{2n(1-E[p])(1-E[e_1])}\Bigg[\frac{h_v}{P}\Big[\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} +$$

$$\frac{n^2\{E[p](1-E[e_2])+ (1-E[p])E[e_1]\}^2}{E[A^2]}\Big] + \frac{h_b}{D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right]\Bigg] \quad (5.24)$$

The second derivative of the equation 5.20 with respect to Q

$$\frac{\partial^2 ETC(n,Q,B_3)}{\partial Q^2} = 2\left[S_v + K + nF + \frac{n(b+h_b)B_3^2}{2D}\right] * \frac{DE[A]}{nQ^3(1-E[p])(1-E[e_1])} \quad (5.25)$$

5.6.2.1 Testing convexity of the cost function

Values of (1 –E[p]), (1 –E[e_1]) are positive because 0 < E[p] < 1, 0 < E[e_1] < 1, 0 < E[e_2] < 1, $B_3 = \frac{h_b\,Q}{(b+h_b)} > 0$ and other variable are positive. This implies $\frac{\partial^2 ETC(n,Q,B_3)}{\partial Q^2} > 0$, $\frac{\partial^2 ETC(n,Q,B_3)}{\partial {B_3}^2} > 0$ and $\frac{\partial^2 ETC(n,Q,B_3)}{\partial Q^2} * \frac{\partial^2 ETC(n,Q,B_3)}{\partial {B_3}^2} -$

$\left[\frac{\partial ETC^2(n,Q,B_3)}{\partial B_3 \partial Q}\right]^2 > 0$ indicating that curve for total cost for all values of Q is Strictly convex and there exists a global (only one) minimum cost for a value of Q (say Q^*) [Please refer table 2.1 of research methodology] . By equating first derivative equal to zero, value of the Q* can be calculated.

$$-[S_v + K + nF]\frac{DE[A]}{nQ^2(1-E[p])(1-E[e_1])} - \frac{{h_b}^2 E[A]}{2(b+h_b)(1-E[p])(1-E[e_1])} +$$

$$\frac{DE[A]}{2n(1-E[p])(1-E[e_1])}\left[\frac{h_v}{P}\left[(2n - n^2) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \right.\right.$$

$$\left.\left.\frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right] + \frac{h_b}{D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right]\right] = 0$$

$$\frac{h_v D}{2P}\left[(2n - n^2) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right] +$$

$$\frac{h_b}{2}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right] - \frac{n{h_b}^2}{2(b+h_b)} = [S_v + K + nF] * \frac{D}{Q^2}$$

$$Q^2 = \frac{[S_v+K+nF]\,D}{\frac{h_v D}{2P}\left[(2n - n^2) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right] + \frac{h_b}{2}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right] - \frac{n{h_b}^2}{2(b+h_b)}}$$

$$Q^* = \sqrt{\frac{[S_v+K+nF]\,D}{\frac{h_v D}{2P}\left[(2n - n^2) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right] + \frac{h_b}{2}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right] - \frac{n{h_b}^2}{2(b+h_b)}}} \quad (5.26)$$

Where $E[A] = 1 - \{E[p](1 - E[e_2]) + (1 - E[p])E[e_1]\}$

$A = 1 - \{p(1 - e_2) + (1 - p)e_1\}$

$A^2 = [1 - \{p(1 - e_2) + (1 - p)e_1\}]^2$

$= 1 - 2p + 2pe_2 - 2e_1 + 2pe_1 + p^2 - 2p^2e_2 + p^2{e_2}^2 + 2pe_1 - 2pe_1e_2 - 2p^2e_1 +$

$2p^2e_1e_2 + {e_1}^2 - 2p{e_1}^2 + p^2{e_1}^2$

Thus

$E[A^2] = 1 - 2E[p]+2E[p]E[e_2] - 2E[e_1] + 2E[p]E[e_1] + E[p^2] -$

$2E[p^2]E[e_2] + E[p^2]E[{e_2}^2] +$

$2E[p]E[e_1] - \quad 2E[p]E[e_1]E[e_2] - 2E[p^2E[e_1] + 2E[p^2]E[e_1]E[e_2] +$

$E[{e_1}^2] - 2E[p]E[{e_1}^2] + E[p^2E[{e_1}^2]$

5.7 The optimal solution for the independent buyer (with back order)

Sometimes the buyer is not interested to work with the vendor for overall cost reduction by adopting integrated vendor-buyer model and looks only for reduction of his/her side of cost only. In such case the buyer tries to optimize his/her cost by conserving only buyer's parameters and order items in single lots. Following model discuss optimization of total cost in buyer's prospective.

Total cost for the buyer and time duration for one complete cycle will be

$$TC_b(Q, B_3) = K + F\ \frac{C_{\alpha\beta} p e_2 Q}{A} + \frac{bB_3^2}{2D} + \frac{h_b}{2D}\left[(Q - B_3)^2 + \frac{(1-p)(1-e_1)pe_2Q^2}{A^2}\right]$$

$$T_c = \frac{(1-p)(1-e_1)Q}{DA}$$

The expected total cost for the buyer and time duration for one complete cycle will be

$$E[TC_b(Q, B_3)] = K + F\ \frac{C_{\alpha\beta} E[p]E[e_2]Q}{E[A]} + \frac{bB_3^2}{2D} + \frac{h_b}{2D}\left[(Q - B_3)^2 + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]Q^2}{E[A^2]}\right]$$

$E[TC_b(Q, B_3)] =$

$$K + F + \frac{C_{\alpha\beta} E[p]E[e_2]Q}{E[A]} + \frac{bB_3^2}{2D} + \frac{h_b}{2D}\left[Q^2 - 2QB_3 + {B_3}^2 + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]Q^2}{E[A^2]}\right] \qquad (5.27)$$

$$E[T_c] = \frac{(1-E[p])(1-E[e_1])Q}{DE[A]}$$

Using renewal and reward theorem [please refer 2.5 of methodology of research and equation 2.4]

$$ETC_b(Q,B_3) = \frac{E[TC_b(Q,B_3)]}{E[T_c]}$$

$$= \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{C_{\alpha\beta}E[p]E[e_2]Q}{E[A]} \text{x} \frac{DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{bB_3^2}{2D} \text{x} \frac{DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{h_bQ^2}{2D} \text{x} \frac{DE[A]}{(1-E[p])(1-E[e_1])Q} -$$

$$\frac{h_bB_3Q}{D} \text{x} \frac{DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{h_bB_3{}^2}{2D} \text{x} \frac{DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{h_b}{2D} \text{x} \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]Q^2}{E[A^2]} \text{x} \frac{DE[A]}{(1-E[p])(1-E[e_1])Q}$$

$$ETC_b(Q,B_3) = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{C_{\alpha\beta}DE[p]E[e_2]E[A]}{(1-E[p])(1-E[e_1])} + \frac{bB_3^2E[A]}{2(1-E[p])(1-E[e_1])Q} + \frac{h_bE[A]Q}{2(1-E[p])(1-E[e_1])} -$$

$$\frac{h_bB_3E[A]}{(1-E[p])(1-E[e_1])} + \frac{h_bB_3{}^2E[A]}{2(1-E[p])(1-E[e_1])Q} + \frac{h_bE[p]E[e_2]Q}{2E[A]} \qquad (5.28)$$

Differentiating equation 5.28 with respect to B_3

$$\frac{\partial ETC_b(Q,B_3)}{\partial B_3} = \frac{bB_3E[A]}{(1-E[p])(1-E[e_1])Q} - \frac{h_bE[A]}{(1-E[p])(1-E[e_1])} + \frac{h_bB_3E[A]}{(1-E[p])(1-E[e_1])Q}$$

$$\frac{\partial ETC_b(Q,B_3)}{\partial B_3} = \frac{(b+h_b)B_3E[A]}{(1-E[p])(1-E[e_1])Q} - \frac{h_bE[A]}{(1-E[p])(1-E[e_1])} \qquad (5.29)$$

Again differentiating equation 5.29 with respect to B_3

$$\frac{\partial ETC_b{}^2(Q,B_3)}{\partial B_3{}^2} = \frac{(b+h_b)E[A]}{(1-E[p])(1-E[e_1])Q} \qquad (5.30)$$

Differentiating (5.29) with respect to Q

$$\frac{\partial ETC_b{}^2(Q,B_3)}{\partial B_3 \partial Q} = -\frac{(b+h_b)B_3E[A]}{(1-E[p])(1-E[e_1])Q^2} \qquad (5.31)$$

Differentiating equation (5.28) with respect to Q

$$\frac{\partial ETC_b(Q,B_3)}{\partial \text{Q}} = -\frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^2} - \frac{bB_3^2E[A]}{2(1-E[p])(1-E[e_1])Q^2} + \frac{h_bE[A]}{2(1-E[p])(1-E[e_1])} -$$

$$\frac{h_bB_3{}^2E[A]}{2(1-E[p])(1-E[e_1])Q^2} + \frac{h_bE[p]E[e_2]}{2E[A]} \qquad (5.32)$$

Again differentiating equation (5.32) with respect to Q

$$\frac{\partial ETC_b{}^2(Q,B_3)}{\partial \text{Q}^2} = +\frac{2(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^3} + \frac{bB_3^2E[A]}{(1-E[p])(1-E[e_1])Q^3} + \frac{h_bB_3{}^2E[A]}{(1-E[p])(1-E[e_1])Q^3} \qquad (5.33)$$

The values of (1-E[p]), (1-E[e_1]) are positive because 0≤E[p] ≤1, 0≤E[e_1] ≤1 and other variables like K, F, D, Q, b, B_3 are also positive. This implies that $\frac{\partial ETC_b{}^2(Q,B_3)}{\partial Q^2} > 0$ (5.33), $\frac{\partial ETC_b{}^2(Q,B_3)}{\partial B_3{}^2} > 0$ (5.30) and $\frac{\partial ETC_b{}^2(Q,B_3)}{\partial Q^2} \text{x} \frac{\partial ETC_b{}^2(Q,B_3)}{\partial B_3{}^2} - \frac{\partial ETC_b{}^2(Q,B_3)}{\partial B_3 \partial Q} > 0$. It indicate that the cost curve $ETC_b(Q, B_3)$ is strictly convex curve and there exists a global (only one) minimum cost for all values of Q. [table 2.1].

Optimal value of B_3 is obtained by putting $\frac{\partial ETC_b(Q,B_3)}{\partial B_3} = 0$, that is

$$\frac{(b+h_b)B_3E[A]}{(1-E[p])(1-E[e_1])Q} - \frac{h_bE[A]}{(1-E[p])(1-E[e_1])} = 0$$

$$\frac{E[A]}{(1-E[p])(1-E[e_1])}\left[\frac{(b+h_b)B_3}{Q} - h_b\right] = 0$$

$$(b + h_b)B_3 = h_bQ$$

$$B_3 = \frac{h_bQ}{(b+h_b)} \qquad (5.34)$$

Putting value of B_3 in equation 5.28

$$ETC_b(Q) = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{C_{\alpha\beta}DE[p]E[e_2]E[A]}{(1-E[p])(1-E[e_1])} + \frac{bE[A]}{2(1-E[p])(1-E[e_1])Q} \text{x} \frac{h_b{}^2Q^2}{(b+h_b)^2} +$$

$$\frac{h_bE[A]Q}{2(1-E[p])(1-E[e_1])} - \frac{h_bE[A]}{(1-E[p])(1-E[e_1])} \text{x} \frac{h_bQ}{(b+h_b)} + \frac{h_bE[A]}{2(1-E[p])(1-E[e_1])Q} \text{x} \frac{h_b{}^2Q^2}{(b+h_b)^2} + \frac{h_bE[p]E[e_2]Q}{2E[A]}$$

$$ETC_b(Q) = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{C_{\alpha\beta}DE[p]E[e_2]E[A]}{(1-E[p])(1-E[e_1])} + \frac{\mathrm{b}h_b{}^2E[A]Q}{2(b+h_b)^2(1-E[p])(1-E[e_1])} +$$

$$\frac{h_bE[A]Q}{2(1-E[p])(1-E[e_1])} - \frac{h_b{}^2E[A]\mathrm{Q}}{(\mathrm{b}+h_b)(1-E[p])(1-E[e_1])} + \frac{h_b{}^3E[A]\mathrm{Q}}{2(b+h_b)^2(1-E[p])(1-E[e_1])} + \frac{h_bE[p]E[e_2]Q}{2E[A]}$$

$$ETC_b(Q) = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{C_{\alpha\beta}DE[p]E[e_2]E[A]}{(1-E[p])(1-E[e_1])} + \frac{(2\mathrm{b}^2+3\mathrm{b}h_b+1)h_bE[A]Q}{2(b+h_b)^2(1-E[p])(1-E[e_1])} +$$

$$+ \frac{h_bE[p]E[e_2]Q}{2E[A]} \qquad (5.35)$$

Differentiating above equation 5.35 with respect to Q

$$\frac{\partial ETC_b(Q)}{\partial Q} = -\frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^2} + \frac{\mathrm{b}{h_b}^2E[A]}{2(b+h_b)^2(1-E[p])(1-E[e_1])} + \frac{h_bE[A]}{2(1-E[p])(1-E[e_1])} -$$

$$\frac{{h_b}^2E[A]}{(\mathrm{b}+h_b)(1-E[p])(1-E[e_1])} + \frac{{h_b}^3E[A]}{2(b+h_b)^2(1-E[p])(1-E[e_1])} + \frac{h_bE[p]E[e_2]}{2E[A]}$$

$$\frac{\partial ETC_b(Q)}{\partial Q} = -\frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^2} + \frac{h_bE[A]}{2(1-E[p])(1-E[e_1])} - \frac{{h_b}^2E[A]}{2(\mathrm{b}+h_b)(1-E[p])(1-E[e_1])} + \frac{h_bE[p]E[e_2]}{2E[A]} \quad (5.36)$$

Optimal value of Q* can be obtained by putting $\frac{\partial ETC_b(Q)}{\partial Q} = 0$.

$$-\frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^2} + \frac{h_bE[A]}{2(1-E[p])(1-E[e_1])} - \frac{{h_b}^2E[A]}{2(\mathrm{b}+h_b)(1-E[p])(1-E[e_1])} + \frac{h_bE[p]E[e_2]}{2E[A]} = 0$$

$$\frac{E[A]}{2(1-E[p])(1-E[e_1])}\left[h_b - \frac{{h_b}^2}{2(\mathrm{b}+h_b)} + \frac{h_bE[p]E[e_2](1-E[p])(1-E[e_1])}{2E[A^2]}\right] = \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q^2}$$

$$h_b - \frac{{h_b}^2}{2(\mathrm{b}+h_b)} + \frac{h_bE[p]E[e_2](1-E[p])(1-E[e_1])}{2E[A^2]} = \frac{(K+F)D}{Q^2}$$

$$h_b\left[2 - \frac{h_b}{(\mathrm{b}+h_b)} + \frac{E[p]E[e_2](1-E[p])(1-E[e_1])}{E[A^2]}\right] = \frac{2(K+F)D}{Q^2}$$

$$h_b\left[1 + \frac{\mathrm{b}}{(\mathrm{b}+h_b)} + \frac{E[p]E[e_2](1-E[p])(1-E[e_1])}{E[A^2]}\right] = \frac{2(K+F)D}{Q^2}$$

$$Q^2 = \frac{2(K+F)D}{h_b\left[1+\frac{\mathrm{b}}{(\mathrm{b}+h_b)}+\frac{E[p]E[e_2](1-E[p])(1-E[e_1])}{E[A^2]}\right]}$$

Thus optimal value of Q* that gives lowest cost for the buyer can be calculated by equation (5.37).

$$Q^* = \sqrt{\frac{2(K+F)D}{h_b\left[1+\frac{\mathrm{b}}{(\mathrm{b}+h_b)}+\frac{E[p]E[e_2](1-E[p])(1-E[e_1])}{E[A^2]}\right]}} \quad (5.37)$$

The total cost of vendor is

$$TC_v(n,Q) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\Big[\left(2n - n^2\right)Q^2 +$$

$$\frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\Big]$$

$$\mathrm{TC_v(Q)} = \mathrm{S_v} + \frac{\mathrm{c_wpQ}}{\mathrm{A}} + \frac{\mathrm{C_r(1-p)e_1Q}}{\mathrm{A}} + \frac{\mathrm{C_{av}pe_2Q}}{\mathrm{A}} + \frac{\mathrm{C_iQ}}{\mathrm{A}} + \frac{\mathrm{h_v}}{\mathrm{2P}}\left[\mathrm{Q^2} + \frac{\mathrm{\{p(1-e_2)+(1-p)e_1\}^2Q^2}}{\mathrm{A^2}}\right]$$

$$E[TC_v(Q)] = S_v + \frac{c_w E[p]Q}{E[A]} + \frac{C_r(1-E[p])E[e_1]Q}{E[A]} + \frac{C_{av}E[p]E[e_2]Q}{E[A]} + \frac{C_i Q}{E[A]} + \frac{h_v}{2P}\left[Q^2 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2 Q^2}{E[A^2]}\right]$$

$$E[TC_v(Q)] = S_v + \{c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\}\frac{Q}{E[A]} + \frac{h_v}{2P}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right]Q^2$$

$$ETC_v(Q) = \frac{E[TC_v(Q)}{E[T]} \quad \text{where } E[T] = \frac{(1-E[p])(1-E[e_1])Q}{DE[A]}$$

$$ETC_v(Q) = S_v * \frac{DE[A]}{(1-E[p])(1-E[e_1])Q} + \{c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\}\frac{Q}{E[A]} * \frac{DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{h_v}{2P}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right]Q^2 * \frac{DE[A]}{(1-E[p])(1-E[e_1])Q}$$

$$ETC_v(Q) = \frac{S_v DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{D\{c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\}}{(1-E[p])(1-E[e_1])} + \frac{h_v D}{2P(1-E[p])(1-E[e_1])}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A]}\right]Q \qquad (5.38)$$

From equation (5.35) and (5.38) total cost will be

$$\text{Total cost } ETC(Q) = ETC_b(Q) + ETC_v(Q)$$

$$= \frac{(K+F)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{C_{\alpha\beta}DE[p]E[e_2]E[A]}{(1-E[p])(1-E[e_1])} + \frac{(2b^2+3bh_b+1)h_b E[A]Q}{2(b+h_b)^2(1-E[p])(1-E[e_1])} + \frac{h_b E[p]E[e_2]Q}{2E[A]} + \frac{S_v DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{D\{c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\}}{(1-E[p])(1-E[e_1])} + \frac{h_v D}{2P(1-E[p])(1-E[e_1])}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A]}\right]Q$$

$$= \frac{(K+F+S_v)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{D\{c_{\alpha\beta}E[p]E[e_2] + c_w E[p] + C_r(1-E[p])E[e_1] + C_{av}E[p]E[e_2] + C_i\}}{(1-E[p])(1-E[e_1])} + \frac{(2b^2+3bh_b+1)h_b E[A]Q}{2(b+h_b)^2(1-E[p])(1-E[e_1])} + \frac{h_b E[p]E[e_2]Q}{2E[A]} + \frac{h_v D}{2P(1-E[p])(1-E[e_1])}\left[1 + \frac{\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A]}\right]Q \qquad (5.39)$$

5.8 Numerical and sensitivity analysis

Considering the integrated vendor- buyer inventory system, where inspection is being performed at vendor site, following parameters are taken. These parameters are also used by (Salamesh & Jaber, 2000), (Wee at al., 2007), (Maddah and Jaber, 2008), (Hsu & Hsu, 2012a) and (Hsu & Hsu, 2012b) for their numerical analysis and cross analysis of their results.

Production rate,	P	= 160,000 units/year
Demand rate,	D	= 50,000 units/year
Inspection rate,	x	=175,200 units/year
Setup cost for vendor,	S_v	= $300/ production run
Ordering cost for buyer,	K	= $100/ order
Holding cost for vender,	h_v	= $2/unit/year
Holding cost for buyer,	h_b	= $5/unit/year
Freight (transportation) cost,	F	= $25/delivery
Inspection cost,	C_i	= $0.5/unit
The cost of producing a defective item,	C_w	= $50/unit
The cost of rejecting a non-defective item,	C_r	= $100/unit
The buyer's post-sales failure cost	$C_{\alpha\beta}$	= $200/unit
The vendor's post-sales failure cost	C_{av}	= $300/unit
The backordering cost	b	= $10/unit/year

The defective percentages p, e_1 and e_2 follow uniform distribution with

$$f(p) = \begin{cases} \frac{1}{\beta}, & 0 \le p \le 0 \\ 0, & otherwise \end{cases} \qquad f(e_1) = \begin{cases} \frac{1}{\lambda}, & 0 \le e_1 \le 0 \\ 0, & otherwise \end{cases} \qquad f(e_2) = \begin{cases} \frac{1}{\eta}, & 0 \le e_2 \le 0 \\ 0, & otherwise \end{cases}$$

$$E[p] = \int_0^\beta pf(p)dy = \int_0^\beta \frac{p}{\beta}dy = \frac{\beta}{2} \qquad E[p^2] = \int_0^\beta p^2 f(p)dy = \int_0^\beta \frac{p^2}{\beta}dy = \frac{\beta^2}{3}$$

$$E[e_1] = \frac{\lambda}{2} \quad E[{e_1}^2] = \frac{\lambda^2}{3} \qquad E[e_2] = \frac{\eta}{2} \qquad E[{e_2}^2] = \frac{\eta^2}{3}$$

If $\beta = \lambda = \eta = 0.04$ then $E[p] = 0.02 \quad E[p^2] = 0.000533333333,\ E[e_1] =$ $0.02 \quad E[{e_1}^2] = 0.000533333333$ and $E[e_2] = 0.02 \quad E[{e_2}^2] = 0.000533333333$

5.8.1 Minimum Expected Total Cost and its comparison

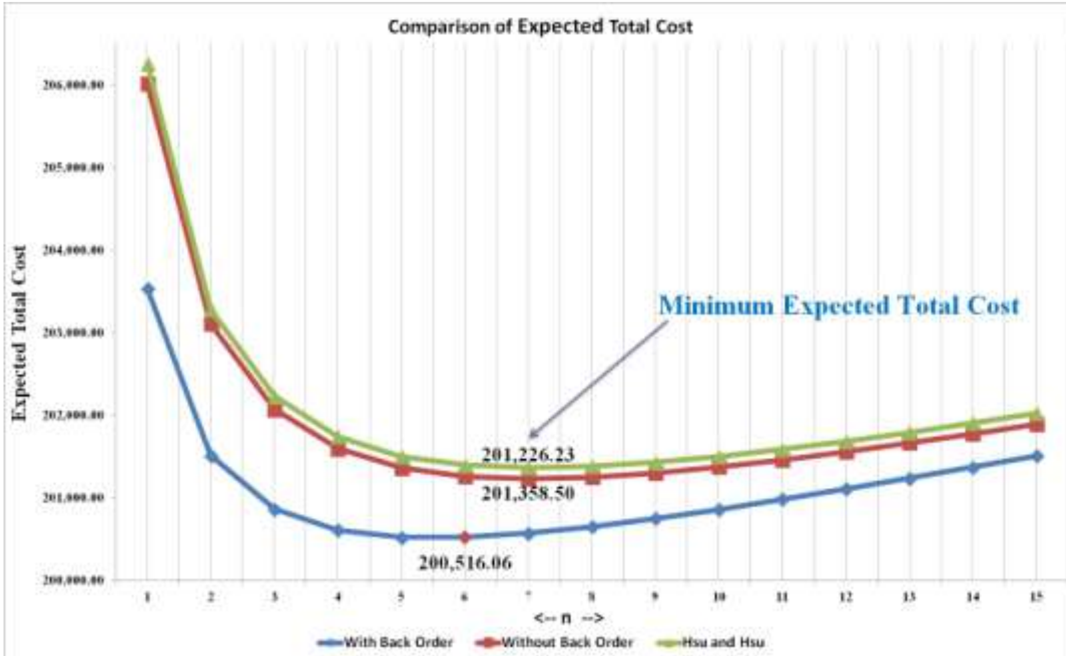
Numerical result for Expected Total Cost with respect to n where P=160,000 units per year, D=50,000 units per year, S_v=\$300 per production run, x=\$175200 units per unit, K=\$100 per order, h_v=\$2 per unit per year, h_b=\$5 per unit per year, F=\$25 per delivery, c_i=\$0.5 per unit, c_w=\$50 per unit, c_r=\$100 per unit, $c_{\alpha\beta}$=\$200 per unit, c_{av}=\$300 unit, $\beta = \lambda = \eta = 0.04$ and comparison of result with (Hsu & Hsu, 2012b)

Table 5.1

n	Result of Numerical Analysis with backorder (this model)				Result of Numerical Analysis Without backorder		(Hsu & Hsu, 2012) Result	
	Q*(n)	**No of Shipments to meet demand D**	**Backorder allowed**	**ETC (n, Q*(n))**	**Q*(n)**	**ETC (n, Q*(n))**	**Q*(n)**	**ETC (n,Q*(n))**
1	3,275.4214	15.27	1,091.8071	203,526.4270	2,748.8152	206,013.2477	2817.4942	206251.9011
2	2,053.5193	24.35	6,84.5065	201,506.9573	1,792.8573	203,100.6283	1839.4721	203281.7266
3	1,536.0641	32.55	5,12.0214	200,857.6005	1,374.9640	202,065.8515	1411.6533	202224.2429
4	1,243.3722	40.21	4,14.4574	200,603.0857	1,132.2595	201,590.0722	1163.0292	201736.5636
5	1,053.5009	47.46	351.1670	200,516.5456	971.4818	201,358.3343	998.2423	201497.8012
6	**919.7634**	**54.36**	**306.5878**	**200,516.0609**	856.3243	201,254.7279	880.1603	201389.8054
7	820.1831	60.96	273.3944	200,564.9501	**769.4031**	**201,226.2280**	**790.9983**	**201358.5041**
8	742.9908	67.30	247.6636	200,644.1344	701.2609	201,245.0781	721.0770	201375.5820
9	681.2941	73.39	227.0980	200,742.8911	646.2796	201,295.3409	664.6448	201424.7759
10	630.7799	79.27	210.2600	200,854.6204	600.8994	201,367.2258	618.0560	201496.0917
11	588.6080	84.95	196.2027	200,975.1313	562.7494	201,454.4000	578.8818	201583.0628
12	552.8299	90.44	184.2766	201,101.7416	530.1867	201,552.5982	545.4389	201681.3327
13	522.0632	95.77	174.0211	201,232.5264	502.0355	201,658.8523	516.5219	201787.8688
14	495.2996	100.95	165.0999	201,366.0628	477.4311	201,771.0413	491.2447	201900.5034
15	471.7860	105.98	157.2620	201,501.4469	455.7232	201,887.6156	468.9401	202017.6520
The Buyer's Independent decision								
n	**Q***	**No of Shipments to meet demand D**	**ETC_b (Q^*,B_3^*)**	**ETC_v (Q^*,B_3^*)**	**ETC (Q*,B3)**	**B_3***		
1	1224.59	40.83	12676.41	199018.22	211694.62	408.20		

Figure 5.3

Figure 5.4



The table 5.3 shows that expected total cost of integrated vendor-buyer with backorder (this model) is \$842.4165 less than that of (Hsu & Hsu, 2012b) and \$710.1431 less than that of the model without backorder. It also shows that 919.7633 are shipped per lot that is larger than that of 769.4031 (without backorder) and 790.9983 (Hsu & Hsu, 2012b) in 6 number of lots in place of 7 lots.

5.8.2 Sensitivity Analysis with respect to F (freight cost) with backorder

Numerical result for Expected Total Cost for integrated solution with respect to different F where P=160,000 units per year, D=50,000 units per year, S_v=\$300 per production run, x=\$175200 units per unit, K=\$100 per order, h_v=\$2 per unit per year, h_b=\$5 per unit per year, F=\$25 per delivery, c_i=\$0.5 per unit, c_w=\$50 per unit, c_r=\$100 per unit, $c_{\alpha\beta}$=\$200 per unit, c_{av}=\$300 unit, $\beta = \lambda = \eta =$ 0.04

Table 5.2

F		Buyer's Independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*,B_3*)	ETC_v (Q_b*,B_3*)	ETC (Q_b*,B3)	B_3*	n*	Q*(n*)	No of Shipments to meet demand D	ETC (n*,Q_b*,B_3*)	
5	1122.36	11965.83	200102.42	212068.25	374.12	12	448.15	111.57	199102.86	12965.38
10	1148.77	12149.40	199803.29	211952.69	382.92	9	603.24	82.89	199574.67	12378.02
15	1174.59	12328.85	199524.24	211853.09	391.53	7	768.64	65.05	199935.29	11917.80
20	1199.85	12504.44	199263.16	211767.60	399.95	6	894.33	55.91	200240.33	11527.27
25	**1224.59**	**12676.41**	**199018.22**	**211694.62**	**408.20**	**6**	**919.76**	**54.36**	**200516.06**	**11178.56**
30	1248.84	12844.97	198787.84	211632.81	416.28	5	1078.29	46.37	200751.19	10881.62
35	1272.63	13010.32	198570.67	211580.99	424.21	5	1102.53	45.35	200980.55	10600.44
40	1295.99	13172.63	198365.51	211538.14	432.00	4	1315.86	38.00	201189.44	10348.70
45	1318.93	13332.07	198171.30	211503.38	439.64	4	1339.15	37.34	201377.84	10125.53
50	1341.47	13488.79	197987.13	211475.92	447.16	4	1362.05	36.71	201563.02	9912.89
55	1363.65	13642.91	197812.17	211455.08	454.55	4	1384.56	36.11	201745.14	9709.94
60	1385.47	13794.57	197645.69	211440.26	461.82	4	1406.72	35.54	201924.35	9515.91
65	1406.95	13943.87	197487.05	211430.92	468.98	3	1719.18	29.08	202086.90	9344.02
70	1428.11	14090.93	197335.65	211426.59	476.04	3	1740.71	28.72	202231.47	9195.11
75	1448.96	14235.84	197190.98	211426.83	482.99	3	1761.99	28.38	202374.28	9052.55
80	1469.51	14378.70	197052.57	211431.27	489.84	3	1783.00	28.04	202515.38	8915.88
85	1489.78	14519.59	196919.97	211439.56	496.59	3	1803.78	27.72	202654.84	8784.72
90	1509.78	14658.58	196792.81	211451.40	503.26	3	1824.32	27.41	202792.71	8658.69
95	1529.51	14795.76	196670.74	211466.50	509.84	3	1844.62	27.11	202929.05	8537.45
100	1549.00	14931.19	196553.43	211484.62	516.33	3	1864.71	26.81	203063.90	8420.72

Figure 5.5

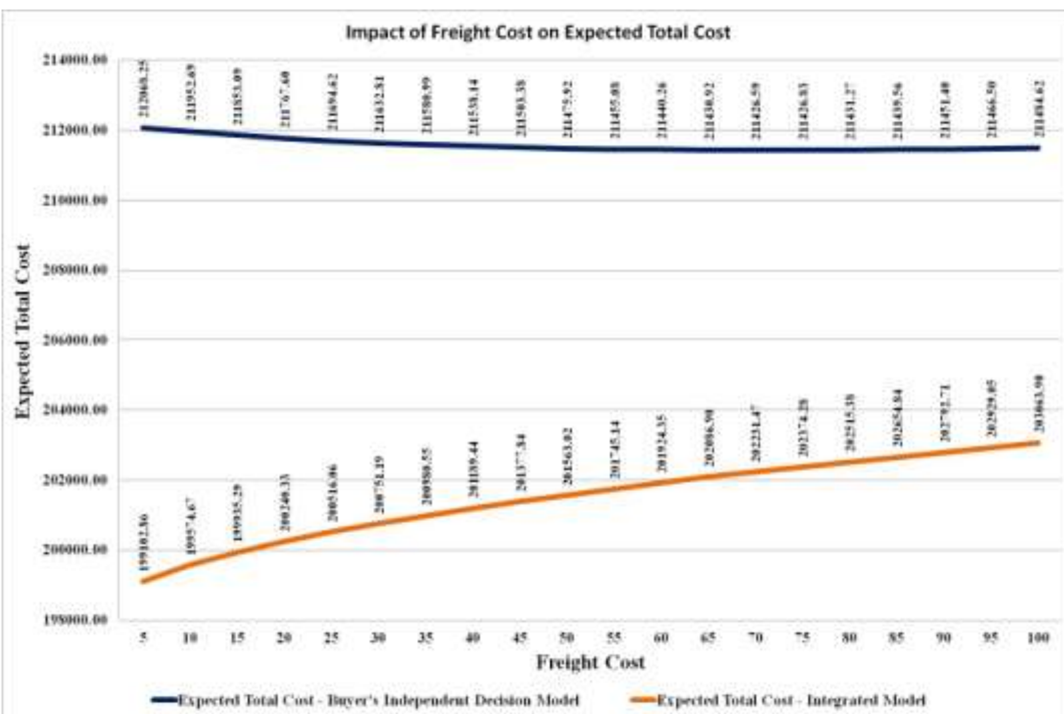


Figure 5.6

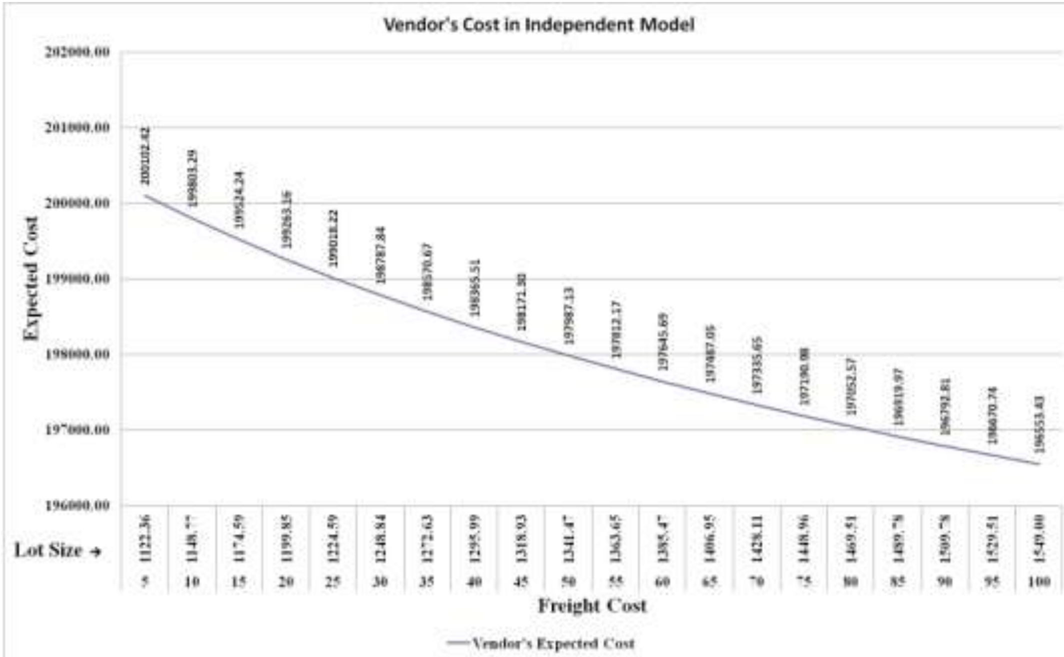
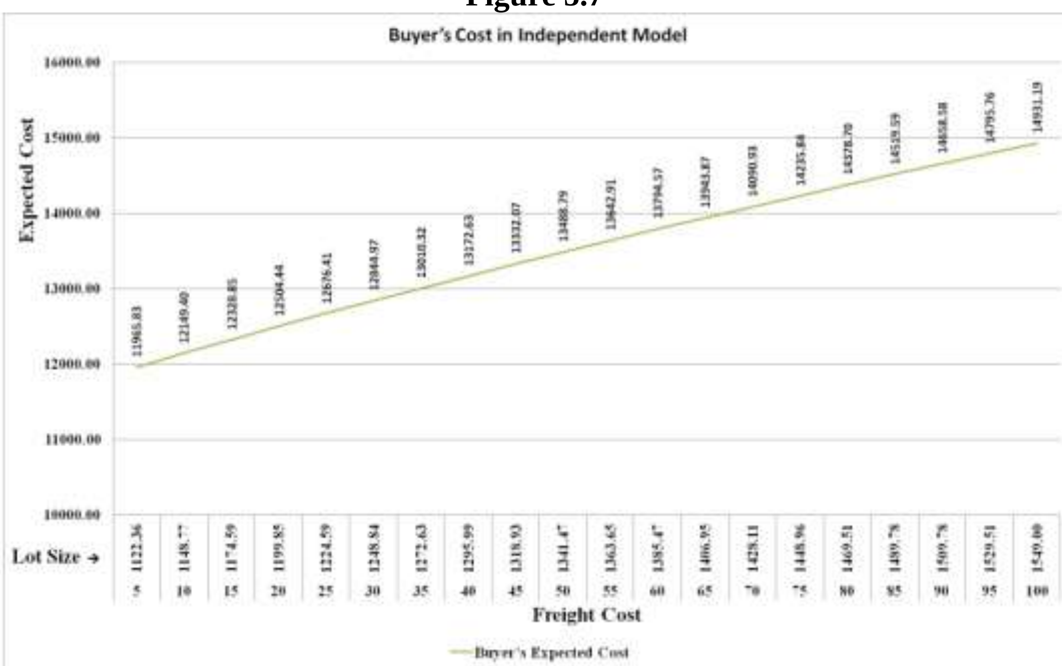


Figure 5.7



The table 5.3 shows that expected total cost of integrated vendor-buyer with backorder (this model) is $842.4165 less than that of (Hsu & Hsu, 2012b) and $710.1431 less than that of the model without backorder. It also shows that

919.7633 are shipped per lot that is larger than that of 769.4031 (without backorder) and 790.9983 (Hsu & Hsu, 2012b) in 6 number of lots in place of 7 lots.

5.8.3 Sensitivity Analysis with respect to h_v (Vendor's Inventory holding cost) with backorder

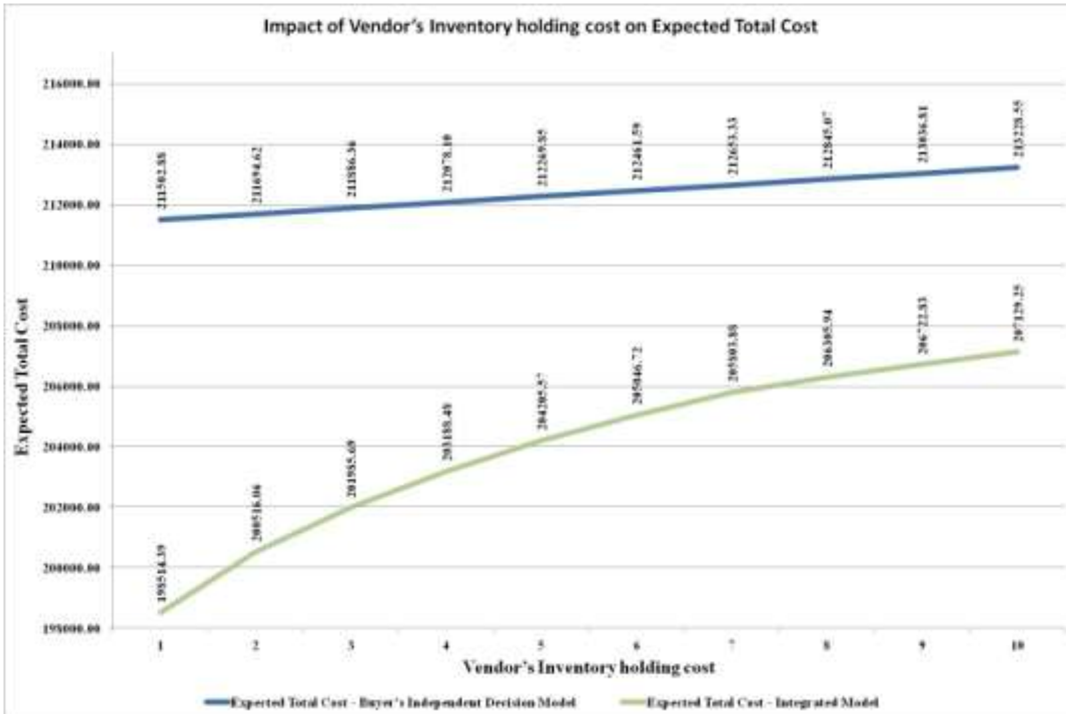
Numerical result for Expected Total Cost for integrated solution with respect to different h_v where P=160,000 units per year, D=50,000 units per year, S_v=\$300 per production run, x=\$175200 units per unit, K=\$100 per order, h_b=\$5 per unit per year, F=\$25 per delivery, c_i=\$0.5 per unit, c_w=\$50 per unit, c_r=\$100 per unit, $c_{\alpha\beta}$=\$200 per unit, c_{av}=\$300 unit, $\beta = \lambda = \eta = 0.04$

Table 5.3

h_v		Buyer's Independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*,B_3*)	ETC_v (Q_b*,B_3*)	ETC (Q_b*,B3)	B_3*	n*	Q*(n*)	No of Shipments to meet demand D	ETC (n*,Q_b*,B_3*)	
1	1224.59	12676.41	198826.48	211502.88	408.20	8	941.56	53.10	198514.39	12988.50
2	**1224.59**	**12676.41**	**199018.22**	**211694.62**	**408.20**	**6**	**919.76**	**54.36**	**200516.06**	**11178.56**
3	1224.59	12676.41	199209.96	211886.36	408.20	4	1093.10	45.74	201985.69	9900.67
4	1224.59	12676.41	199401.70	212078.10	408.20	3	1252.87	39.91	203188.48	8889.63
5	1224.59	12676.41	199593.44	212269.85	408.20	3	1159.59	43.12	204205.57	8064.27
6	1224.59	12676.41	199785.18	212461.59	408.20	2	1552.25	32.21	205046.72	7414.87
7	1224.59	12676.41	199976.92	212653.33	408.20	2	1475.22	33.89	205803.88	6849.44
8	1224.59	12676.41	200168.66	212845.07	408.20	1	2697.76	18.53	206305.94	6539.12
9	1224.59	12676.41	200360.40	213036.81	408.20	1	2628.24	19.02	206722.83	6313.98
10	1224.59	12676.41	200552.14	213228.55	408.20	1	2563.83	19.50	207129.25	6099.30

The table 5.3 shows Expected Total Cost for buyer's independent and integrated solution for different vendor's holding cost. It is observed as vendor's holding cost increased the number of lots per production batch from the vendor to the buyer is decreased and size of lots is also increased. The cost reduction of integrated model from buyer’s independent decision is higher for smaller vendor’s holding cost and it decreased as vendor’s holding cost increased.

Figure 5.8



5.8.4 Sensitivity Analysis with respect to h_b (Buyer’s Inventory holding cost)

Numerical result for Expected Total Cost for integrated solution respect to different the buyer’s inventory holding cost h_b per unit item where P=160,000 units per year, D=50,000 units per year, S_v=\$300 per production run, x=\$175200 units per unit, K=\$100 per order, h_v=\$2 per unit per year, F=\$25 per delivery, c_i=\$0.5 per unit, c_w=\$50 per unit, c_r=\$100 per unit, $c_{\alpha\beta}$=\$200 per unit, c_{av}=\$300 unit, $\beta = \lambda = \eta = 0.04$

Table 5.4

h_b		Buyer's Independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*,B_3*)	ETC_v (Q_b*,B_3*)	ETC (Q_b*,B3)	B_3*	n*	Q*(n*)	No of Shipments to meet demand D	ETC (n*,Q_b*,B_3*)	
1	2558.55	8830.96	193047.00	201877.96	232.60	1	5260.93	9.50	198627.37	3250.59
2	1846.16	10374.98	195087.13	205462.11	307.69	3	1771.84	28.22	199485.40	5976.71
3	1534.44	11375.43	196640.77	208016.20	354.10	4	1330.65	37.58	199943.43	8072.76
4	1349.99	12106.24	197919.22	210025.47	385.71	5	1081.06	46.25	200262.32	9763.15
5	**1224.59**	**12676.41**	**199018.22**	**211694.62**	**408.20**	**6**	**919.76**	**54.36**	**200516.06**	**11178.56**
6	1132.13	13141.52	199990.04	213131.56	424.55	6	902.57	55.40	200705.98	12425.57
7	1060.21	13533.76	200866.70	214400.46	436.56	6	888.18	56.29	200870.66	13529.80
8	1002.10	13873.27	201669.33	215542.60	445.38	7	785.23	63.68	201010.94	14531.66
9	953.79	14173.39	202412.63	216586.02	451.79	7	776.75	64.37	201125.19	15460.83
10	912.74	14443.28	203107.28	217550.56	456.37	7	769.35	64.99	201226.97	16323.59

Figure 5.9

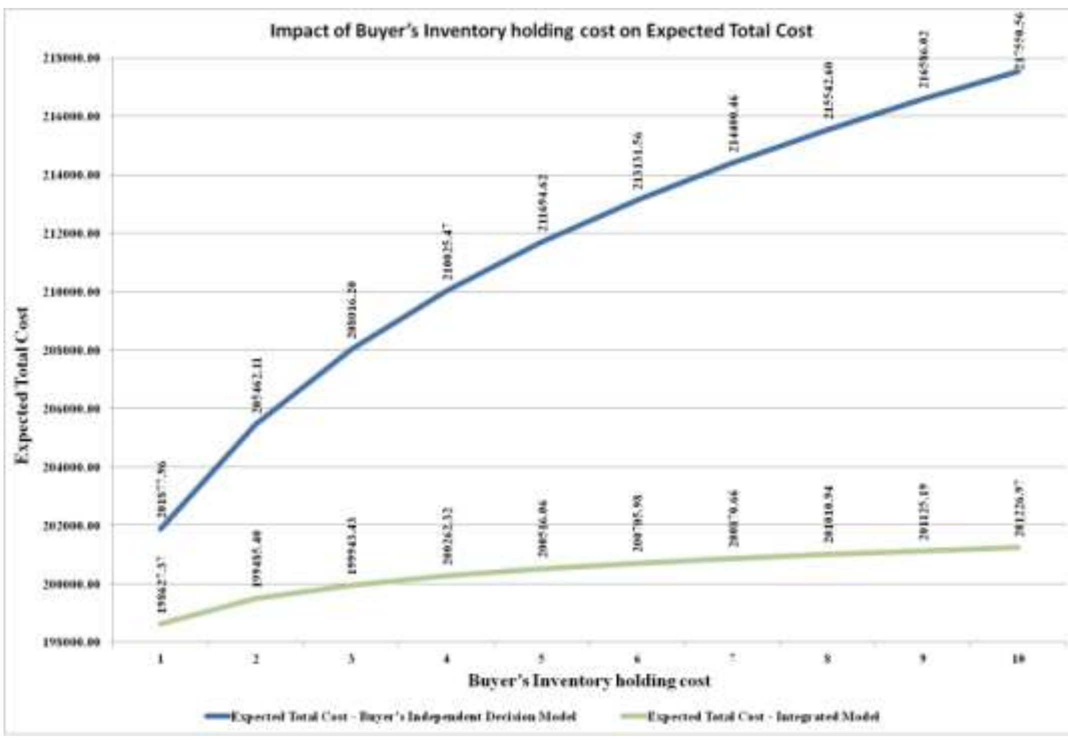


Table 5.4 shows Expected Total Cost for buyer's independent and integrated solution for different buyer's holding cost. It is observed that the impact of buyer's holding cost is just opposite to the vendor's holding cost. As the buyer's holding cost increased the number of lots per production batch from the vendor to the buyer is also increasing and size of lots is decreasing. The cost reduction of integrated model from buyer's independent decision is lesser for smaller buyer's holding cost and it increases as buyer's holding cost is increasing.

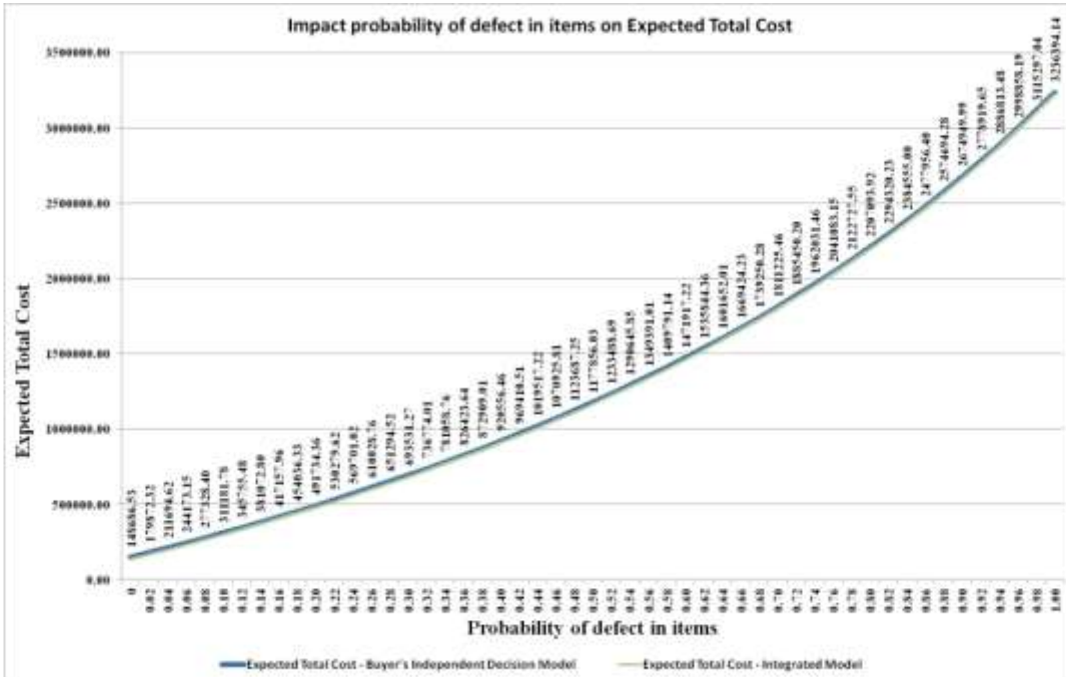
5.8.5 Sensitivity Analysis with respect to different defective percentage

Numerical result for Expected Total Cost for integrated solution with respect to different probability of defective percentage β where P=160,000 units per year, D=50,000 units per year, S_v=\$300 per production run, x=\$175200 units per unit, K=\$100 per order, h_v=\$2 per unit per year, h_b=\$5 per unit per year, F=\$25 per delivery, c_i=\$0.5 per unit, c_w=\$50 per unit, c_r=\$100 per unit, $c_{\alpha\beta}$=\$200 per unit, c_{av}=\$300 unit, $\lambda = \eta = 0.04$

Table 5.5

β		Buyer's Independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*,B_3*)	ETC_v (Q_b*,B_3*)	ETC (Q_b*,B3)	B_3*	n*	Q*(n*)	No of Shipments to meet demand D	ETC (n*,Q_b*,B_3*)	
0.00	1224.74	8505.17	140181.36	148686.53	408.25	6	919.80	54.36	137516.95	11169.58
0.02	1224.67	10569.72	169302.59	179872.32	408.22	6	919.80	54.36	168698.16	11174.16
0.04	**1224.59**	**12676.41**	**199018.22**	**211694.62**	**408.20**	**6**	**919.76**	**54.36**	**200516.06**	**11178.56**
0.06	1224.51	14826.53	229346.62	244173.15	408.17	6	919.69	54.37	232990.38	11182.77
0.08	1224.43	17021.44	260306.96	277328.40	408.14	6	919.59	54.37	266141.73	11186.68
0.10	1224.35	19262.56	291919.23	311181.78	408.12	6	919.44	54.38	299991.29	11190.49
0.12	1224.27	21551.35	324204.12	345755.48	408.09	6	919.25	54.39	334561.53	11193.95
0.14	1224.18	23889.37	357183.43	381072.80	408.06	6	919.01	54.41	369875.65	11197.15
0.16	1224.10	26278.21	390879.75	417157.96	408.03	6	918.73	54.42	405957.92	11200.04
0.18	1224.01	28719.55	425316.78	454036.33	408.00	6	918.40	54.44	442833.70	11202.63
0.20	1223.92	31215.14	460519.22	491734.36	407.97	6	918.02	54.47	480529.44	11204.92
0.22	1223.83	33766.79	496512.83	530279.62	407.94	6	917.59	54.49	519072.68	11206.94
0.24	1223.74	36376.44	533324.58	569701.02	407.91	5	1050.50	47.60	558492.46	11208.56
0.26	1223.64	39046.07	570982.68	610028.76	407.88	5	1049.88	47.62	598818.88	11209.88
0.28	1223.54	41777.78	609516.74	651294.52	407.85	5	1049.20	47.66	640083.60	11210.92
0.30	1223.45	44573.75	648957.51	693531.27	407.82	5	1048.44	47.69	682319.92	11211.35
0.32	1223.35	47436.30	689337.71	736774.01	407.78	5	1047.60	47.73	725562.43	11211.58
0.34	1223.24	50367.80	730690.96	781058.76	407.75	5	1046.68	47.77	769847.36	11211.40
0.36	1223.14	53370.79	773052.85	826423.64	407.71	5	1045.68	47.82	815213.20	11210.44
0.38	1223.04	56447.92	816461.09	872909.01	407.68	5	1044.59	47.87	861699.87	11209.14
0.40	1222.93	59601.96	860954.50	920556.46	407.64	5	1043.41	47.92	909349.35	11207.11
0.42	1222.82	62835.84	906574.67	969410.51	407.61	5	1042.13	47.98	958205.63	11204.88
0.44	1222.71	66152.61	953364.61	1019517.22	407.57	5	1040.76	48.04	1008315.52	11201.71
0.46	1222.60	69555.51	1001370.30	1070925.81	407.53	5	1039.27	48.11	1059727.68	11198.13
0.48	1222.49	73047.94	1050639.31	1123687.25	407.50	5	1037.68	48.18	1112493.68	11193.57
0.50	1222.37	76633.49	1101222.54	1177856.03	407.46	5	1035.98	48.26	1166667.20	11188.83
0.52	1222.25	80315.91	1153172.77	1233488.69	407.42	5	1034.15	48.35	1222305.79	11182.89
0.54	1222.13	84099.19	1206546.66	1290645.85	407.38	5	1032.21	48.44	1279469.14	11176.70
0.56	1222.01	87987.52	1261403.49	1349391.01	407.34	5	1030.14	48.54	1338221.57	11169.44
0.58	1221.89	91985.36	1317805.78	1409791.14	407.30	5	1027.94	48.64	1398629.89	11161.25
0.60	1221.77	96097.38	1375819.84	1471917.22	407.26	5	1025.60	48.75	1460764.66	11152.56
0.62	1221.65	100328.55	1435515.81	1535844.36	407.22	5	1023.13	48.87	1524701.57	11142.79
0.64	1221.52	104684.14	1496967.87	1601652.01	407.17	5	1020.52	48.99	1590519.56	11132.44
0.66	1221.39	109169.67	1560254.56	1669424.23	407.13	5	1017.76	49.13	1658303.21	11121.02
0.68	1221.27	113791.10	1625459.18	1739250.28	407.09	5	1014.85	49.27	1728141.83	11108.45
0.70	1221.14	118554.67	1692670.78	1811225.46	407.05	5	1011.80	49.42	1800130.24	11095.22
0.72	1221.01	123467.03	1761983.17	1885450.20	407.00	5	1008.59	49.57	1874369.00	11081.20
0.74	1220.88	128535.30	1833496.17	1962031.46	406.96	5	1005.22	49.74	1950965.44	11066.02
0.76	1220.75	133766.96	1907316.20	2041083.15	406.92	5	1001.69	49.92	2030033.37	11049.78
0.78	1220.62	139170.09	1983557.46	2122727.55	406.87	5	998.01	50.10	2111694.74	11032.81
0.80	1220.49	144753.25	2062340.67	2207093.92	406.83	5	994.16	50.29	2196078.60	11015.32
0.82	1220.36	150525.57	2143794.65	2294320.23	406.79	5	990.16	50.50	2283324.02	10996.21
0.84	1220.24	156496.87	2228058.13	2384555.00	406.75	5	985.99	50.71	2373578.06	10976.95
0.86	1220.11	162677.58	2315278.82	2477956.40	406.70	5	981.67	50.93	2467000.01	10956.39
0.88	1219.98	169078.93	2405615.35	2574694.28	406.66	5	977.19	51.17	2563758.69	10935.58
0.90	1219.86	175712.91	2499237.07	2674949.99	406.62	5	972.55	51.41	2664036.64	10913.35
0.92	1219.74	182592.52	2596327.13	2778919.65	406.58	5	967.76	51.67	2768028.39	10891.26
0.94	1219.62	189731.57	2697081.91	2886813.48	406.54	5	962.83	51.93	2875944.99	10868.50
0.96	1219.50	197145.11	2801713.08	2998858.19	406.50	5	957.75	52.21	2988012.45	10845.74
0.98	1219.38	204849.22	2910447.82	3115297.04	406.46	5	952.54	52.49	3104474.30	10822.74
1.00	1219.27	212861.32	3023532.82	3236394.14	406.42	5	947.20	52.79	3225595.05	10799.09

Figure 5.10



The table 5.5 shows Expected Total Cost for buyer's independent and integrated solution for different defective percentage β where defective percentage is uniformly distributed between 0 and β. As β increases the cost reduction decreased and Expected Total Cost in both situations increased.

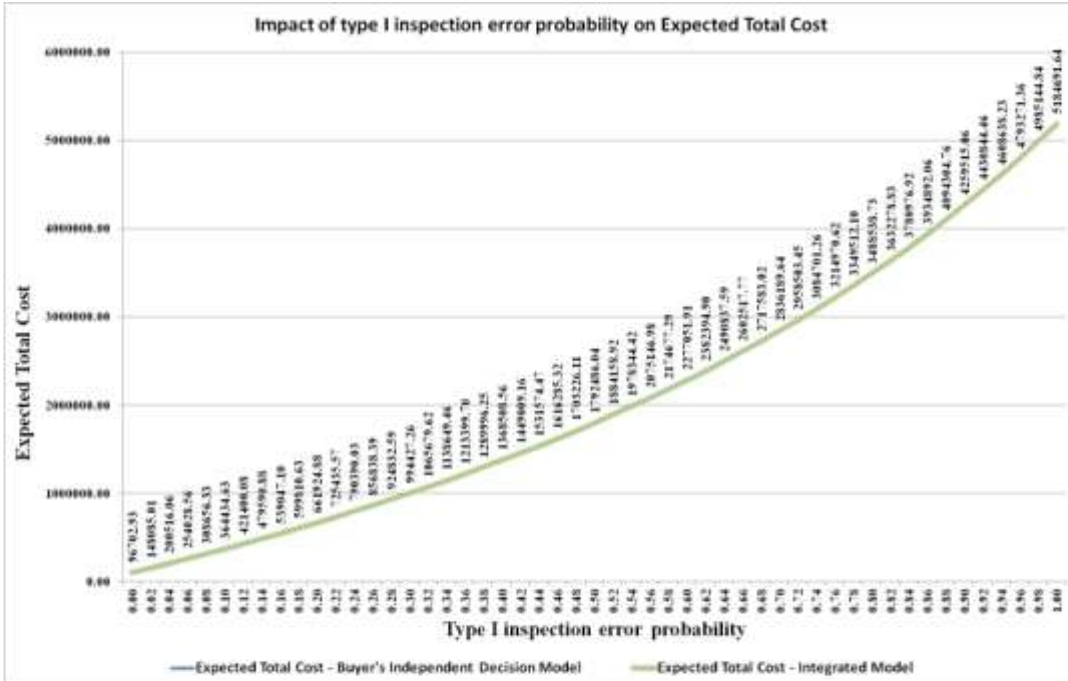
5.8.6 Sensitivity Analysis with respect to different values of type I inspection error percentage e_1

Numerical result for Expected Total Cost for integrated solution with respect to different type I inspection error probability λ where P=160,000 units per year, D=50,000 units per year, S_v=\$300 per production run, x=\$175200 units per unit, K=\$100 per order, h_v=\$2 per unit per year, h_b=\$5 per unit per year, F=\$25 per delivery, c_i=\$0.5 per unit, c_w=\$50 per unit, c_r=\$100 per unit, $c_{\alpha\beta}$=\$200 per unit, c_{av}=\$300 unit, $\beta = \eta = 0.04$

Table 5.6

λ		Buyer's Independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*,B_3*)	ETC_v (Q_b*,B_3*)	ETC (Q_b*,B3)	B_3*	n*	Q*(n*)	No of Shipments to meet demand D	ETC (n*,Q_b*,B_3*)	
0.00	1224.60	12592.98	95290.00	107882.98	408.20	6	919.96	54.35	96702.93	11180.05
0.02	1224.59	12634.27	146630.19	159264.47	408.20	6	919.88	54.35	148085.01	11179.45
0.04	**1224.59**	**12676.41**	**199018.22**	**211694.62**	**408.20**	**6**	**919.76**	**54.36**	**200516.06**	**11178.56**
0.06	1224.59	12719.41	252486.51	265205.92	408.20	6	919.61	54.37	254028.56	11177.36
0.08	1224.59	12763.31	307068.84	319832.15	408.20	6	919.41	54.38	308656.33	11175.82
0.10	1224.59	12808.13	362800.31	375608.43	408.20	6	919.17	54.40	364434.63	11173.81
0.12	1224.59	12853.90	419717.70	432571.60	408.20	6	918.89	54.41	421400.08	11171.52
0.14	1224.58	12900.66	477859.18	490759.84	408.19	5	1052.18	47.52	479590.88	11168.96
0.16	1224.58	12948.44	537264.75	550213.19	408.19	5	1051.76	47.54	539047.10	11166.09
0.18	1224.58	12997.26	597976.03	610973.29	408.19	5	1051.28	47.56	599810.63	11162.66
0.20	1224.58	13047.17	660036.57	673083.74	408.19	5	1050.74	47.59	661924.88	11158.86
0.22	1224.58	13098.20	723491.82	736590.02	408.19	5	1050.13	47.61	725435.57	11154.46
0.24	1224.58	13150.39	788389.42	801539.82	408.19	5	1049.46	47.64	790390.03	11149.78
0.26	1224.57	13203.78	854779.01	867982.79	408.19	5	1048.72	47.68	856838.39	11144.40
0.28	1224.57	13258.41	922712.78	935971.19	408.19	5	1047.89	47.71	924832.59	11138.59
0.30	1224.57	13314.33	992244.99	1005559.31	408.19	5	1047.00	47.76	994427.26	11132.05
0.32	1224.57	13371.57	1063433.09	1076804.66	408.19	5	1046.01	47.80	1065679.62	11125.04
0.34	1224.57	13430.20	1136336.79	1149766.99	408.19	5	1044.94	47.85	1138649.46	11117.53
0.36	1224.57	13490.25	1211018.46	1224508.72	408.19	5	1043.78	47.90	1213399.70	11109.02
0.38	1224.56	13551.79	1287544.64	1301096.43	408.19	5	1042.53	47.96	1289996.25	11100.18
0.40	1224.56	13614.87	1365983.83	1379598.70	408.19	5	1041.17	48.02	1368508.56	11090.14
0.42	1224.56	13679.54	1446409.30	1460088.84	408.19	5	1039.71	48.09	1449009.16	11079.68
0.44	1224.56	13745.86	1528897.06	1542642.93	408.19	5	1038.15	48.16	1531574.47	11068.45
0.46	1224.56	13813.91	1613527.65	1627341.56	408.19	5	1036.47	48.24	1616285.32	11056.25
0.48	1224.55	13883.75	1700385.45	1714269.21	408.18	5	1034.67	48.32	1703226.11	11043.09
0.50	1224.55	13955.46	1789559.86	1803515.32	408.18	5	1032.75	48.41	1792486.04	11029.28
0.52	1224.55	14029.10	1881144.41	1895173.51	408.18	5	1030.71	48.51	1884158.92	11014.59
0.54	1224.55	14104.75	1975238.25	1989343.00	408.18	5	1028.53	48.61	1978344.42	10998.59
0.56	1224.55	14182.51	2071946.29	2086128.79	408.18	5	1026.22	48.72	2075146.98	10981.81
0.58	1224.55	14262.45	2171378.79	2185641.24	408.18	5	1023.78	48.84	2174677.29	10963.95
0.60	1224.54	14344.68	2273652.42	2287997.10	408.18	5	1021.19	48.96	2277051.91	10945.19
0.62	1224.54	14429.29	2378890.85	2393320.14	408.18	5	1018.45	49.09	2382394.90	10925.23
0.64	1224.54	14516.38	2487224.86	2501741.25	408.18	5	1015.56	49.23	2490837.59	10903.66
0.66	1224.54	14606.08	2598793.49	2613399.57	408.18	5	1012.52	49.38	2602517.77	10881.80
0.68	1224.54	14698.49	2713742.54	2728441.03	408.18	5	1009.32	49.54	2717583.02	10858.02
0.70	1224.53	14793.74	2832229.56	2847023.31	408.18	5	1005.96	49.70	2836189.64	10833.67
0.72	1224.53	14891.97	2954419.86	2969311.83	408.18	5	1002.43	49.88	2958503.45	10808.38
0.74	1224.53	14993.32	3080489.48	3095482.80	408.18	5	998.74	50.06	3084701.26	10781.54
0.76	1224.53	15097.93	3210626.25	3225724.18	408.18	5	994.89	50.26	3214970.62	10753.55
0.78	1224.53	15205.97	3345030.48	3360236.45	408.18	5	990.87	50.46	3349512.10	10724.35
0.80	1224.53	15317.61	3483915.51	3499233.11	408.18	5	986.68	50.67	3488538.73	10694.38
0.82	1224.53	15433.03	3627508.91	3642941.93	408.18	5	982.32	50.90	3632278.83	10663.11
0.84	1224.53	15552.43	3776054.77	3791607.20	408.18	5	977.80	51.14	3780976.92	10630.27
0.86	1224.52	15676.01	3929812.95	3945488.96	408.17	5	973.12	51.38	3934892.06	10596.90
0.88	1224.52	15804.01	4089063.55	4104867.56	408.17	5	968.27	51.64	4094304.76	10562.80
0.90	1224.52	15936.65	4254105.94	4270042.60	408.17	5	963.26	51.91	4259515.06	10527.54
0.92	1224.52	16074.21	4425262.02	4441336.23	408.17	5	958.10	52.19	4430844.46	10491.77
0.94	1224.52	16216.96	4602877.41	4619094.37	408.17	5	952.79	52.48	4608638.23	10456.13
0.96	1224.52	16365.19	4787325.78	4803690.98	408.17	5	947.34	52.78	4793271.36	10419.61
0.98	1224.52	16519.23	4979007.82	4995527.06	408.17	5	941.75	53.09	4985144.84	10382.22
1.00	1224.52	16679.44	5178358.22	5195037.66	408.17	5	936.04	53.42	5184691.64	10346.02

Figure 5.11



The table 5.6 shows Expected Total Cost for buyer's independent and integrated solution for different values of type I inspection error percentage e_1 which is uniformly distributed between 0 and λ. As λ increases the cost reduction decreased and Expected Total Cost in both situations increased rapidly.

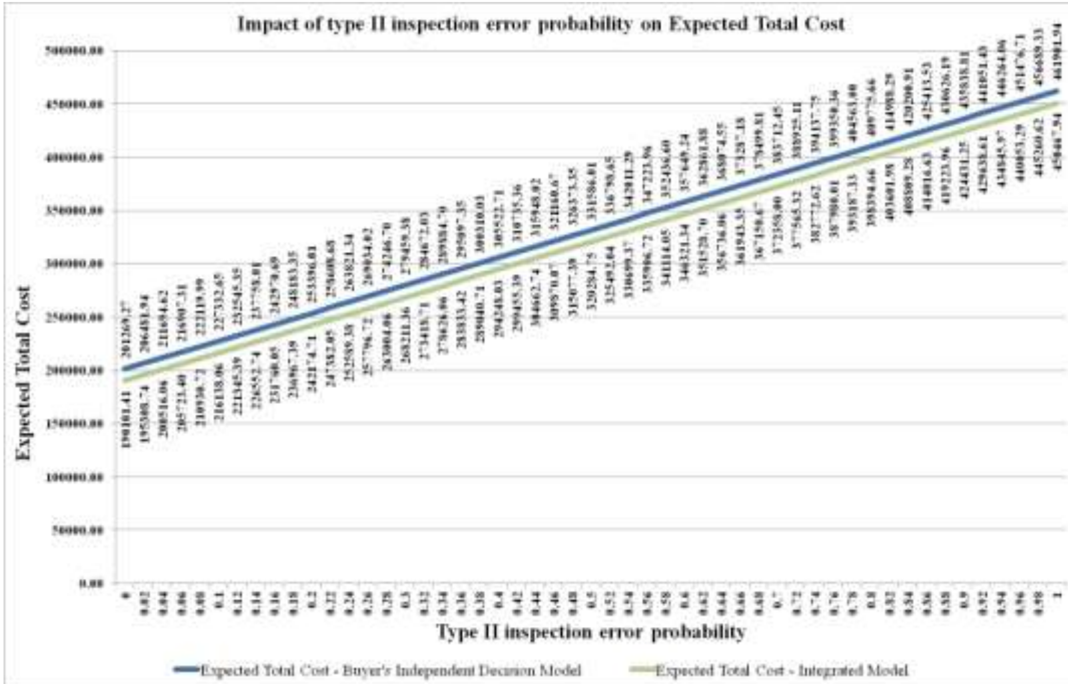
5.8.7 Sensitivity Analysis with respect to different values of type II inspection error e_2

Numerical result for Expected Total Cost for integrated solution with respect to type II inspection error probability η where P=160,000 units per year, D=50,000 units per year, S_v=\$300 per production run, x=\$175200 units per unit, K=\$100 per order, h_v=\$2 per unit per year, h_b=\$5 per unit per year, F=\$25 per delivery, c_i=\$0.5 per unit, c_w=\$50 per unit, c_r=\$100 per unit, $c_{\alpha\beta}$=\$200 per unit, c_{av}=\$300 unit, $\beta = \lambda = 0.04$

Table 5.7

η		Buyer's Independent decision				Integrated model				Cost Reduction in Integrated Model
	Q_b*	ETC_b (Q_b*,B_3*)	ETC_v (Q_b*,B_3*)	ETC (Q_b*,B3)	B_3*	n*	Q*(n*)	No of Shipments to meet demand D	ETC (n*,Q_b*,B_3*)	
0.00	1224.74	8505.17	192764.10	201269.27	408.25	6	919.60	54.37	190101.41	11167.86
0.02	1224.67	10590.79	195891.15	206481.94	408.22	6	919.68	54.37	195308.74	11173.20
0.04	**1224.59**	**12676.41**	**199018.22**	**211694.62**	**408.20**	**6**	**919.76**	**54.36**	**200516.06**	**11178.56**
0.06	1224.52	14762.02	202145.28	216907.31	408.17	6	919.85	54.36	205723.40	11183.91
0.08	1224.44	16847.64	205272.35	222119.99	408.15	6	919.93	54.35	210930.72	11189.26
0.10	1224.36	18933.25	208399.40	227332.65	408.12	6	920.01	54.35	216138.06	11194.59
0.12	1224.29	21018.87	211526.48	232545.35	408.10	6	920.10	54.34	221345.39	11199.96
0.14	1224.21	23104.48	214653.53	237758.01	408.07	6	920.18	54.34	226552.74	11205.27
0.16	1224.14	25190.09	217780.60	242970.69	408.05	6	920.27	54.33	231760.05	11210.64
0.18	1224.06	27275.70	220907.65	248183.35	408.02	6	920.35	54.33	236967.39	11215.96
0.20	1223.98	29361.31	224034.70	253396.01	407.99	6	920.43	54.32	242174.71	11221.30
0.22	1223.91	31446.92	227161.76	258608.68	407.97	6	920.52	54.32	247382.05	11226.63
0.24	1223.83	33532.53	230288.81	263821.34	407.94	6	920.60	54.31	252589.38	11231.97
0.26	1223.76	35618.15	233415.87	269034.02	407.92	6	920.68	54.31	257796.72	11237.30
0.28	1223.68	37703.75	236542.95	274246.70	407.89	6	920.77	54.30	263004.06	11242.65
0.30	1223.61	39789.36	239670.01	279459.38	407.87	6	920.85	54.30	268211.36	11248.01
0.32	1223.53	41874.97	242797.06	284672.03	407.84	6	920.93	54.29	273418.71	11253.32
0.34	1223.46	43960.58	245924.12	289884.70	407.82	6	921.02	54.29	278626.06	11258.64
0.36	1223.38	46046.18	249051.17	295097.35	407.79	6	921.10	54.28	283833.42	11263.94
0.38	1223.31	48131.79	252178.25	300310.03	407.77	6	921.18	54.28	289040.71	11269.33
0.40	1223.23	50217.40	255305.31	305522.71	407.74	6	921.27	54.27	294248.03	11274.67
0.42	1223.16	52303.00	258432.36	310735.36	407.72	6	921.35	54.27	299455.39	11279.97
0.44	1223.08	54388.60	261559.42	315948.02	407.69	6	921.43	54.26	304662.74	11285.27
0.46	1223.01	56474.21	264686.46	321160.67	407.67	6	921.52	54.26	309870.07	11290.60
0.48	1222.93	58559.81	267813.54	326373.35	407.64	6	921.60	54.25	315077.39	11295.95
0.50	1222.86	60645.41	270940.60	331586.01	407.62	6	921.68	54.25	320284.75	11301.26
0.52	1222.78	62731.01	274067.64	336798.65	407.59	6	921.77	54.24	325492.04	11306.61
0.54	1222.71	64816.61	277194.69	342011.29	407.57	6	921.85	54.24	330699.37	11311.92
0.56	1222.63	66902.20	280321.76	347223.96	407.54	6	921.93	54.23	335906.72	11317.24
0.58	1222.56	68987.80	283448.80	352436.60	407.52	6	922.02	54.23	341114.05	11322.55
0.60	1222.48	71073.39	286575.85	357649.24	407.49	6	922.10	54.22	346321.34	11327.90
0.62	1222.41	73158.99	289702.89	362861.88	407.47	6	922.18	54.22	351528.70	11333.18
0.64	1222.34	75244.58	292829.96	368074.55	407.45	6	922.27	54.21	356736.06	11338.49
0.66	1222.26	77330.18	295957.00	373287.18	407.42	6	922.35	54.21	361943.35	11343.83
0.68	1222.19	79415.77	299084.05	378499.81	407.40	6	922.43	54.20	367150.67	11349.14
0.70	1222.11	81501.36	302211.09	383712.45	407.37	6	922.51	54.20	372358.00	11354.45
0.72	1222.04	83586.95	305338.16	388925.11	407.35	6	922.60	54.19	377565.32	11359.78
0.74	1221.97	85672.55	308465.20	394137.75	407.32	6	922.68	54.19	382772.62	11365.13
0.76	1221.89	87758.13	311592.24	399350.36	407.30	6	922.76	54.19	387980.01	11370.36
0.78	1221.82	89843.72	314719.28	404563.00	407.27	6	922.85	54.18	393187.33	11375.67
0.80	1221.75	91929.30	317846.35	409775.66	407.25	6	922.93	54.18	398394.66	11381.00
0.82	1221.67	94014.90	320973.39	414988.29	407.22	6	923.01	54.17	403601.98	11386.31
0.84	1221.60	96100.48	324100.43	420200.91	407.20	6	923.10	54.17	408809.28	11391.63
0.86	1221.53	98186.07	327227.47	425413.53	407.18	6	923.18	54.16	414016.63	11396.90
0.88	1221.45	100271.66	330354.54	430626.19	407.15	6	923.26	54.16	419223.96	11402.24
0.90	1221.38	102357.24	333481.58	435838.81	407.13	6	923.34	54.15	424431.25	11407.56
0.92	1221.31	104442.82	336608.61	441051.43	407.10	6	923.43	54.15	429638.61	11412.82
0.94	1221.23	106528.41	339735.65	446264.06	407.08	6	923.51	54.14	434845.97	11418.09
0.96	1221.16	108613.99	342862.72	451476.71	407.05	6	923.59	54.14	440053.29	11423.42
0.98	1221.09	110699.58	345989.76	456689.33	407.03	6	923.67	54.13	445260.62	11428.71
1.00	1221.02	112785.15	349116.79	461901.94	407.01	6	923.76	54.13	450467.94	11434.00

Figure 5.12



The table 5.7 shows Expected Total Cost for buyer's independent and integrated solution for different values of type II inspection error percentage e_2 which is uniformly distributed between 0 and η. As η increases Expected Total Cost in both situations increased.

CHAPTER 6

Findings, Results, Conclusions, Implications and Scope for Future Work

6.1 Introduction

The Research work by (Goyal, 1977) and (Banerjee, 1986) had opened a new area of research in the field of Supply Chain, Inventory Management. Thereafter, there a number of research works had done that extends the single product, single vendor and single buyer with a lot of diversification with practical aspects. National Semiconductor, Wal-Mart, Procter and Gamble and many more organizations have benefitted from these research works.

As per the research gap identified, this research work tried to analyze the impact on expected total cost of the inventory management when inspection process has been shifted from the buyer's place to the vendor place. The inspection has been conduced along with production of items. For these models production process has been assumed imperfect and there are some defective items in production lots. There has been a 100% inspection of items produced. The inspection process is also assumed imperfect. There are type I and type II inspection errors. Using these assumptions following models have been discussed in this research work

- Integrated model where backorder has not been allowed
- The Buyers independent decision where backorder has not been allowed
- Integrated model where backorder has been allowed
- The Buyers independent decision where backorder has been allowed

6.2 Result Analysis of the model where backorder is not allowed

6.2.1 Integrated model where backorder has not allowed: The vendor and the buyer work together and tried to reduce total inventory carrying cost. For the purpose the buyer place order for some quantity, the vendor produces those quantities in a single production lot, inspect items for defects and supply n lots to the buyer at a fixed interval. In this case backorder has been not allowed. The economic order quantity (EOQ), Q*, for the model has been derived as

$$Q^* = \sqrt{\frac{[S_v+K+nF\ ]}{\frac{h_v}{2P}\left\{(2n-n^2)+\frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]}+\frac{n^2\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A^2]}\right\}+\frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}}}$$

Where $A = 1 - \{p(1-e_2) + (1-p)e_1\}$

$E[A] = 1 - \{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}$

$E[A^2] = 1 - 2E[p]+2E[p]E[e_2] - 2E[e_1] + 2E[p]E[e_1] + E[p^2] - 2E[p^2]E[e_2] + E[p^2]E[{e_2}^2] + 2E[p]E[e_1] - 2E[p]E[e_1]E[e_2] - 2E[p^2E[e_1] + 2E[p^2]E[e_1]E[e_2] + E[{e_1}^2] - 2E[p]E[{e_1}^2] + E[p^2E[{e_1}^2]$

The Expected Total Cost, ETC(n,Q) of the integrated model without backorder has been derived as which depends up on two variables n (number of lots per order) and Q (quantity of items in a lot).

$$ETC(n,Q) = [S_v + K + nF\ ] * \frac{DE[A]}{n(1-E[p])(1-E[e_1])Q} + \Big[nc_wE[p] + nC_r(1-E[p])E[e_1] + nC_{av}E[p]E[e_2] + nC_i + nc_{\alpha\beta}E[p]E[e_2]\Big] * \frac{D}{n(1-E[p])(1-E[e_1])} + \Big[\frac{h_v}{2P}\Big\{(2n-n^2) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A^2]}\Big\} + \frac{nh_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}]\,(1-E[p])(1-E[e_1])}{2E[A^2]D}\Big] Q * \frac{DE[A]}{n(1-E[p])(1-E[e_1])}$$

6.2.2 The Buyer's independent decision where backorder has not been allowed: The buyer and the vendor does not work together, the buyer take decisions to optimize his inventory cost and ignoring all cost parameters of the vendor that are affecting total cost of the inventory management. In this model, after optimizing inventory cost, the vendor place order for Q quantity, gets supply of q items in a single lot and do not allow any backorder. For this model Economic Order Quantity (EOQ) has been derived as

$$Q^* = \sqrt{\frac{2(K+F)DE[A^2]}{h_b[1-\{E[p](1-2E[e_2])+(1-E[p])E[e_1]\}](1-E[p])(1-E[e_1])}}$$

The Expected Total Cost, ETC(Q) of the buyer's independent decision model without backorder has been derived as which depends up on Q (quantity of items in a lot).

$$\mathrm{ETC(Q)} =$$

$$\frac{(\mathrm{K+F+S_v})\mathrm{DE[A]}}{(1-\mathrm{E[p]})(1-\mathrm{E[e_1]})\mathrm{Q}} + \frac{\mathrm{D}\{\mathrm{c_{\alpha\beta}E[p]E[e_2]+c_wE[p]+C_r(1-E[p])E[e_1]+C_{av}E[p]E[e_2]+C_i}\}}{(1-\mathrm{E[p]})(1-\mathrm{E[e_1]})} +$$

$$\frac{\mathrm{h_b}}{2}\frac{[1-\{\mathrm{E[p](1-2E[e_2])+(1-E[p])E[e_1]}\}]\mathrm{Q}}{\mathrm{E[A]}} + \frac{\mathrm{h_{v\,D}}}{2\mathrm{P}(1-\mathrm{E[p]})(1-\mathrm{E[e_1]})}\left[1+\frac{\{\mathrm{E[p](1-E[e_2])+(1-E[p])E[e_1]}\}^2}{\mathrm{E[A]}}\right]\mathrm{Q}$$

6.2.3 Integrated model where backorder has been allowed:

The situation is same as mentioned in 6.1.1except that backorder (shortage in inventory) has been allowed with consent from the buyer. There is a cost due to shortage of items like goodwill, sale loss etc but at the same time there are other factors like less inventory level that could reduce the overall expected total cost of the inventory management. The Economic Order Quantity (EOQ), Q* for the model has been derived as

$$Q^* = \sqrt{\frac{[S_v+K+nF]\,D}{\frac{h_v D}{2P}\left[(2n-n^2)+\frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]}+\frac{n^2\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A^2]}\right]+\frac{h_b}{2}\left[n+\frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right]-\frac{n{h_b}^2}{2(b+h_b)}}}$$

Where $A = 1-\{p(1-e_2)+(1-p)e_1\}$

$E[A] = 1-\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}$

$E[A^2] = 1-2E[p]+2E[p]E[e_2]-2E[e_1]+2E[p]E[e_1]+E[p^2]-2E[p^2]E[e_2]+E[p^2]E[{e_2}^2]+2E[p]E[e_1]-2E[p]E[e_1]E[e_2]-2E[p^2E[e_1]+2E[p^2]E[e_1]E[e_2]+E[{e_1}^2]-2E[p]E[{e_1}^2]+E[p^2E[{e_1}^2]$

Optimal backorder quantity B_3* for the model has been derived as

$$B_3{}^* = \frac{h_b Q^*}{(b + h_b)}$$

The Expected Total Cost, ETC(n,Q) of the integrated model with backorder has been derived as which depends up on two variables n (number of lots per order) and Q (quantity of items in a lot).

$$ETC(n,Q) = [S_v + K + nF] * \frac{DE[A]}{nQ(1-E[p])(1-E[e_1])} + \Big[nc_w E[p] + nC_r(1 - E[p])E[e_1] + nC_{av}E[p]E[e_2] \;\; + nC_i + nc_{\alpha\beta}E[p]E[e_2]\Big] * \frac{D}{n(1-E[p])(1-E[e_1])} - \frac{{h_b}^2 E[A]Q}{2(b+h_b)(1-E[p])(1-E[e_1])} + \frac{DQE[A]}{2n(1-E[p])(1-E[e_1])}\left[\frac{h_v}{P}\left[\left(2n - n^2\right) + \frac{n(n-1)P(1-E[p])(1-E[e_1])}{DE[A]} + \frac{n^2\{E[p](1-E[e_2]) + (1-E[p])E[e_1]\}^2}{E[A^2]}\right] + \frac{h_b}{D}\left[n + \frac{(1-E[p])(1-E[e_1])E[p]E[e_2]}{E[A^2]}\right]\right]$$

6.2.4 The Buyers independent decision where backorder has been allowed:

As mentioned in 6.1.2, the buyer may take its own decision based on his/her own parameters. In this buyer's independent decision model where the buyer allow backorder in the inventory, the Economic Order Quantity (EOQ) has been derived as

$$Q^* = \sqrt{\frac{2(K+F)D}{h_b\left[1 + \frac{\mathrm{b}}{(\mathrm{b}+h_b)} + \frac{E[p]E[e_2](1-E[p])(1-E[e_1])}{E[A^2]}\right]}}$$

Optimal backorder quantity B_3* for the model has been derived as

$$B_3{}^* = \frac{h_b Q^*}{(b+h_b)}$$

The Expected Total Cost, ETC(Q) of the buyer's independent decision model without backorder has been derived as which depends up on Q (quantity of items in a lot).

$$\mathrm{ETC(Q)} = \frac{(K+F+S_v)DE[A]}{(1-E[p])(1-E[e_1])Q} + \frac{D\{c_{\alpha\beta}E[p]E[e_2]+c_w E[p]+C_r(1-E[p])E[e_1]+C_{av}E[p]E[e_2]+C_i\}}{(1-E[p])(1-E[e_1])} + \frac{b{h_b}^2E[A]Q}{2(b+h_b)^2(1-E[p])(1-E[e_1])} + \frac{h_bE[A]Q}{2(1-E[p])(1-E[e_1])} - \frac{{h_b}^2E[A]Q}{(b+h_b)(1-E[p])(1-E[e_1])} + \frac{{h_b}^3E[A]Q}{2(b+h_b)^2(1-E[p])(1-E[e_1])} + \frac{h_bE[p]E[e_2]Q}{2E[A]} + \frac{h_vD}{2P(1-E[p])(1-E[e_1])}\left[1+\frac{\{E[p](1-E[e_2])+(1-E[p])E[e_1]\}^2}{E[A]}\right]Q$$

6.3 Result and Discussions

6.3.1 Integrated model where backorder has not been allowed

The vendor and the buyer work in coordination in the integrated model without backorder. The model has been tested with a numerical example. The parameter values taken in the numerical example have same values as taken by earlier models which enable to compare results between models.

Form the numerical example, the minimum Expected Total Cost (ETC) is calculated to 2,01,226.23$ for integrated model where backorder has been not allowed. The ETC value is less than ETC value 2,01,358.50$ calculated by (Hsu & Hsu, 2012b) model which was minimum among earlier researches.

The economic lot size has calculated to 769.4031 for the integrated model where backorder has been not allowed for the numerical example. The economic lot size is less than 790.9983 items that had derived by the (Hsu & Hsu, 2012b). It indicates that less number of quantities has to be transported to the buyer. The outcome of the numerical example is in the line of assumption of the model where inspection before shipment filtered out defective items leading to reduction of shipment size. The table 4.1 shows a comparison of the economic shipment lot size with (Hsu & Hsu, 2012b).

6.3.2 The Buyers independent decision where backorder has not been allowed

The buyer's take decision independently and tried to optimize his/her costs depending upon cost parameters related to him/her. In this model cost parameters related to the vendor have been totally ignored.

The model is tested by a numerical example with same parameters used in the integrated mode. The minimum Expected Total Cost (ETC) for this model is calculated to 2,08,459.45$ with economic lot size 1581.3539 items per lot (single lot per order). The ETC of the model is higher than ETC, 2,01,226.23$, of the integrated model with same parameter values.

The above finding suggests that for optimization of Expected Total Cost (ETC), all cost factors related to both the vendor and the buyer are important. Hence the buyer and the vendor should work in coordination for reduction of integrated inventory model.

6.3.3 Integrated model where backorder has been allowed:

The vendor and the buyer are working in coordination in the integrated model with backorder. The model is tested with a numerical example. The parameter values used for the model are same with without backorder and backorder related values from earlier models.

Form the numerical example, the minimum Expected Total Cost (ETC) is calculated to 2,00,516.0609$ for the integrated model where controlled backorder has been allowed. The Economic Lot Size (ELS) has been calculated to 919.7634 items per lot and optimal backorder size has calculated to 306.5878 items.

The comparisons of values obtained from integrated models with backorder and without backorder have been given below

Table 6.1 Comparison of integrated models

Values	**Integrated Model without backorder**	**Integrated Model with backorder**
minimum Expected Total Cost (ETC) (in $)	2,01,226.23$	2,00,516.06$
Economic Lot Size (number of items)	769.4	919.8
Optimum Backorder quantity (number of items)	NIL	306.6

The minimum Expected Total Cost (ETC) is less for the integrated model with backorder (Table 6.1). Which indicates that controlled backorder reduced expected total cost of the inventory management. Controlled backorder is good a practice for inventory management.

It is also observed that economic order quantity for the integrated model with backorder is larger than without backorder. The backorder is the number of items that are delivered immediately by the buyer to consumers upon arrival of a fresh lot of items. Number of items, going stored in the buyer's inventory is less. The table 6.1 shows that from 919.8 items 306.6 items are immediately handed over to consumers (backorder) and rest 613.2 (919.8 – 306.6) items are stored in the inventory which is less than 769.4 items of the integrated model without backorder.

6.3.4 The Buyers independent decision where backorder has been allowed

When the buyer's take decision independently, and tried to optimize total cost occurred to the buyer with backorder allowed. A numerical example, using same cost values parameter, calculates the minimum Expected Total Cost (ETC) to 2,11,694.62$ and economic lot size to 1224.59 items per lot. This ETC is higher than the integrated models. (2,00,516.0609$ of when backorder has been allowed and 2,01,226.23$ when backorder is not allowed.)

Table 6.2 Comparison of buyer's independent decision models

Values	Buyer's independent decision model without backorder	Buyer's independent decision model with backorder
minimum Expected Total Cost (ETC) (in $)	208459.45$	2,11,694.62$
Economic Lot Size (number of items)	1581.35	1224.59
Optimum Backorder quantity (number of items)	NIL	408.20

The finding suggests that the buyer and the vendor should work together to minimize expected total cost. Allowing backorder is not a good decision.

6.3.5 Sensitivity Analysis of freight cost

The freight cost is sensitive to minimum expected total cost of the integrated model. Increase in freight cost also increases the minimum expected total cost. But it is not sensitive to ETC of the buyer's independent decision model. Figure 4.4 and figure 5.3 shows the pattern. The integrated model needs one or more shipments to ship ordered items but for independent decision model needs only one shipment. It makes the independent decision model less sensitive. Increase in freight cost leads to increase in the economic lot size and production lot size for the vendor. Increase in production lot size reduces expected total cost to the vendor. Figure 5.4 shows the pattern. This analysis is equally applicable to both without and with backorder.

The expected total cost of integrated model is always less than the buyer's independent decision model.

The finding suggests that the buyer and the vendor should work together to minimize expected total cost. Keeping control on freight cost helps to reduce expected total cost.

6.3.6 Sensitivity Analysis of vendor's inventory holding cost

The vendor's inventory holding cost is very sensitive to the integrated model and less sensitive to buyer's independent decision model. (as depicted in Figure 4.5 and figure 5.6).

In the Buyer's independent model, items are stored in vendor's inventory only during the time of production. When the production process has been completed, all produced items are shipped to the buyer.

In the integrated model, the situation is different. All produced items are stored in the vendor's inventory and are shipped to the buyer in a number of lots, (as depicted in Figure 4.1 and Figure 4.2). As large number of item is stored for a long duration, it becomes sensitive for expected total cost.

The expected total cost of integrated model is always less than the buyer's independent decision model.

The finding suggests that control on the vendors' inventory holding cost is very important to reduce expected total cost and the buyer and the vendor should work together to minimize expected total cost.

6.3.7 Sensitivity Analysis of buyer's inventory cost

The buyer's inventory holding cost is not sensitive to the integrated model and very sensitive to buyer's independent decision model. (Figure 4.6 and figure 5.7).

The expected total cost of integrated model is always less than the buyer's independent decision model.

The finding suggests that the buyer's inventory holding cost is not important to reduce expected total cost and the buyer and the vendor should work together to minimize expected total cost.

6.3.8 Sensitivity Analysis of probability of defects p, type I inspection error percentage e_1 and probability of type II inspection error e_2

The buyer's inventory holding cost is very sensitive to the integrated model and buyer's independent decision model. (Refer to figure 4.7 and figure 5.8) Expected total cost increased exponentially with increase in percentage of imperfect quality p, type I inspection error e_1 and type II inspection error e_2. The values in both the integrated and the buyer's independent models are almost same.

The finding suggests that the percentage of imperfect quality p, type I inspection error e_1 and type II inspection error e_2 should be low to reduce expected total cost.

6.4 Suggestions

1. Inspection of items should be done at the vendor’s site.
2. From the above analysis of models, it is clear that the minimum expected total cost of integrated model is always less than the buyer’s independent decision model. It is suggested that the vendor and buyers should work in cooperation to reduce overall cost.
3. Shortage backorder of inventory should be allowed as it reduces the cost further in case of integrated inventory management. But shortage backorder should not be allowed for buyer’s independent decision for economic order quantity (EOQ).
4. For integrated model inventory carrying cost is very critical. All efforts should be taken care to keep it to the minimum.
5. Probably of all three types of errors should be kept to a minimum. Increase probability of any error (production of defective items, type my inspection error, type II inspection error) has increased the cost exponentially.

6.5 Implications for the Industry

Advancement in technologies and ease in transportation has changed the way supply chain management is working. On-line commerce needs to maintain inventories of products. The research work would ensure industries in taking correct decisions for maintaining their inventories. As suggested, they should go for 100% screening of items on the vendor’s site in place of random checking of items, which has been done for quality control. The research also helps industries to identify areas where they can focus to reduce the expected total cost of their inventory.

6.5.1 Managerial insights

This research work discussed management of two different inventories; the vendor manages one inventory and the buyer manages second inventory. It also provides information about timing of production of items. The objective of the vendor and the buyer is to reduce total expected cost of inventories in the supply chain. Above models show that when the buyer try to reduce his/her cost without taking account of the vendor's cost, total expected cost is high for all cases. This indicates that the buyer must work with the vendor to reduce total expected cost.

Allowing backorder in the buyer's inventory is a tricky decision for management. Due to backorder, they can lose their customers, profit and goodwill. Considering these costs, above model tried to find out an optimal backorder quantity, where profit is bigger than the side effect costs. Table 6.1 shows that a calculated backorder in the integrated vendor-buyer model reduced total expected cost. Management of a manufacturing industry can put their data in the model and get optimal backorder quantity for more benefit.

Analysis of above models indicates that backorder should not be allowed I any case where the buyer is taking independent decision.

6,5.2 Implementation strategy

The above models have been tested with numerical values that were used by researchers. A manufacturing industry can put their recorded values in the formulas derived in above models and get Economic Order Quantities (EOQ), production batch quantity, optimum backorder quantity etc. along with expected total cost of the inventories management.

The cost parameters are not constant. These cost parameters could be reduced with some efforts from the management. The sensitivities analysis of above models will help management to identify those areas where the management could work so that expected total cost get reduce further. For example, rate of defects in production and rate of inspection errors are badly increasing expected total cost. All efforts must be done to reduce and control any types of error (Production and Inspection).

6.6 Limitation and Scope for future work

The buyer is dealing with a number of items. When inspectors are working at the buyer's site, the time they are actually working is very less, but when the inspection process is done at the vendor's site their working time will increase and their idle time will be reduced thereafter reducing the cost of inspection per unit item. This assumption was not covered in this research and could be taken for future work.

Training of inspectors, use of technology and advanced equipment may reduce the inspection cost and reduce probability of type I and type II errors. Total

Quality Management can be used to reduce the probability of having defects in an item. Though, quality management checks for defective items, during the production process, it could not ensure 100% defect-free production process. The impact of quality management and training of inspectors could be taken as a future scope.

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APPENDICES

Appendix A1

LIST OF PUBLICATIONS

- "Integrated Single-Vendor Single-Buyer inventory model with imperfect production and Inspection by vendor", in 2019 Second International Conference on Advanced Computational and Communication Paradigms, held on 25-28 February 2019 (Scopus)
- "Inventory Model for items with imperfect quality and screening at vendor site" in The IUJ Journal of Management, Volume 2, issue 1, May 2014, pp. 97-101, ISSN: 2347-5080
- "Opportunities of cloud computing in supply Chain Management" in Anusandhanika, Volume V, Number I & II, 2013, ISSN 0974-200X

Appendix A2

This section presents code of C programs used in mathematical model "An integrated single-vendor, single-buyer inventory model for imperfect quality production, imperfect inspection at vendor site"

1. Calculation of the minimum Expected Total Cost for the integrated solution with respect to lot size Q and number of lots n.

Program Code: A2.1

```
#include <stdio.h>
#include <math.h>
int main()
{
	float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, bita=0.04 ;
	float q1,q2,q3,q4,q5,qt;
	float El, El2, Ee1, Ee2, Ee12, EA, EA2;
	float q,ETCq,ETCqv;
	float Ep, Ep2, Ee22;
	int n;
```

```
    double A1,A2,A3,A4,A5,A6,A7,A8,A9;
    double B1,B2,B3,B4,B5,B6,bkorder,etc;

    Ep = bita/2;
    Ee1 = bita/2;
    Ee2 = bita/2;

    Ep2 = (bita*bita)/3;
    Ee12 = (bita*bita)/3;
    Ee22 = (bita*bita)/3;

    EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
    EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 + 2*Ep*Ee1
- 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;
    for(n=1;n<=15;n++)
    {
        A1 = (sv + K + n*F);
        A2 = 2*n - n*n;
        A3 = (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA);
        A4 = (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2;
        A5 = ((A2 + A3 + A4) * hv)/(2*P);
        A6 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1)) / (2*EA2*D);
        A7 = A1 / ( A5 + A6);
        q = sqrt(A7);
        B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
        B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-
Ep)*(1-Ee1));

        B3 = (((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-
Ee2)+(1-Ep)*Ee1))/EA2)* hv)/ (2*P);
        B4 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1))/(2*EA2*D);
        B5 = ((B3 +B4)*q*D*EA)/(n*(1-Ep)*(1-Ee1));
        etc = B1 + B2 + B5;
        printf("n = %-4d   Q* = %-14.4f   etc = %-14.4f\n",n,q,etc);
    }
    return 0;
}
```

2. The minimum Expected Total Cost for the integrated solution with respect to different freight cost F.

Program Code: A2.2

```
#include <stdio.h>
#include <math.h>
int main()
{
    float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, bita=0.04, alfa=0.04, neno=0.04;
    float q1,qind,q3,q4,q5,qt,ind_etcV,ind_etcB,ind_etcT,cost_diff;
    float El, El2, Ee1, Ee2, Ee12, EA, EA2;
    float q,ETCq,ETCqv;
    float Ep, Ep2, Ee22;
    int n,opt_n=0;
    double A1,A2,A3,A4,A5,A6,A7,A8,A9;
    double B1,B2,B3,B4,B5,B6,etc;
    double c1,c2,c3,c4,c5,c6,cetc;
```

```
	double opt_q=0.0, opt_etc;

	printf("F     Qb           ETCb(Qb)       ETCv(Qv)       Total Cost      n    Q(n)          ETC(n,Q(n))
Cost Reduction\n");
	for(F=5;F<=100;F=F+5)
	{
		Ep = bita/2;
		Ee1 = alfa/2;
		Ee2 = neno/2;

		//Square
		Ep2 = (bita*bita)/3;
		Ee12 = (alfa*alfa)/3;
		Ee22 = (neno*neno)/3;

		EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
		EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 +
2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;

		opt_n = 0;
		opt_q = 0.0;
		opt_etc = 99999999.0;

		for(n=1;n<=15;n++)
		{
			A1 = (sv + K + n*F);
			A2 = 2*n - n*n;
			A3 = (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA);
			A4 = (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2;
			A5 = ((A2 + A3 + A4) * hv)/(2*P);
			A6 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1)) / (2*EA2*D);
			A7 = A1 / ( A5 + A6);

			q = sqrt(A7);


			B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
			B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-
Ep)*(1-Ee1));
			B3 = (((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-
Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2)* hv)/ (2*P);
			B4 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1))/(2*EA2*D);
			B5 = ((B3 +B4)*q*D*EA)/(n*(1-Ep)*(1-Ee1));
			etc = B1 + B2 + B5;

			if (opt_etc > etc || n == 0 )
			{
				opt_n = n;
				opt_q = q;
				opt_etc = etc;
			}
		}

		//indipendant solution
		c1 = 2*(K + F)*D*EA2;
		c2 = hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1);
		qind = sqrt(c1/c2);

		//Total Cost(Buyer)
```

```
        ind_etcB = (((K + F)*D*EA)/((1-Ep)*(1-Ee1)*qind)) + ((cab*D*Ep*Ee2)/((1-Ep)*(1-Ee1))) +
(hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*qind)/(2*EA);

        //Total Cost(Vendor)
        ind_etcV = ((D/((1-Ep)*(1-Ee1)))*(((sv*EA)/qind) +cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci +
(hv/(2*P))*(1 +((Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA))*qind));
        ind_etcT = ind_etcV + ind_etcB;

    cost_diff = ind_etcT- opt_etc;
        printf("%-3.0f   %-14.2f   %-14.2f   %-14.2f   %-14.2f   %-2d   %-14.2f   %-14.2f   %-
14.2f\n",F,qind,ind_etcB,ind_etcV,ind_etcT, opt_n,opt_q,opt_etc,cost_diff);
    }
    return 0;
    }
```

3. The minimum Expected Total Cost for the integrated solution with respect to different vendor's inventory holding cost h_v.

Program Code: A2.3

```
#include <stdio.h>
#include <math.h>
int main()
{
	float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, bita=0.04, alfa=0.04, neno=0.04;
	float q1,qind,q3,q4,q5,qt,ind_etcV,ind_etcB,ind_etcT,cost_diff;
	float El, El2, Ee1, Ee2, Ee12, EA, EA2;
	float q,ETCq,ETCqv;
	float Ep, Ep2, Ee22;
	int n,opt_n=0;
	double A1,A2,A3,A4,A5,A6,A7,A8,A9;
	double B1,B2,B3,B4,B5,B6,etc;
	double c1,c2,c3,c4,c5,c6,cetc;
	double opt_q=0.0, opt_etc;

	printf("hv     Qb          ETCb(Qb)      ETCv(Qv)      Total Cost     n    Q(n)          ETC(n,Q(n))
Cost Reduction\n");
	for(hv=1;hv<=10;hv++)
	{
		Ep = bita/2;
		Ee1 = alfa/2;
		Ee2 = neno/2;

		//Square
		Ep2 = (bita*bita)/3;
		Ee12 = (alfa*alfa)/3;
		Ee22 = (neno*neno)/3;

		EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
		EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 +
2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;
		opt_n = 0;
		opt_q = 0.0;
		opt_etc = 99999999.0;
		for(n=1;n<=15;n++)
		{
			A1 = (sv + K + n*F);
			A2 = 2*n - n*n;
			A3 = (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA);
			A4 = (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2;
			A5 = ((A2 + A3 + A4) * hv)/(2*P);
			A6 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1)) / (2*EA2*D);
			A7 = A1 / ( A5 + A6);

			q = sqrt(A7);

			B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
			B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-
Ep)*(1-Ee1));
			B3 = (((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-
Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2)* hv)/ (2*P);
			B4 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1))/(2*EA2*D);
			B5 = ((B3 +B4)*q*D*EA)/(n*(1-Ep)*(1-Ee1));
```

```
            etc = B1 + B2 + B5;
            if (opt_etc > etc || n == 0 )
            {
                opt_n = n;
                opt_q = q;
                opt_etc = etc;
            }
        }

        //indipendant solution
        c1 = 2*(K + F)*D*EA2;
        c2 = hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1);
        qind = sqrt(c1/c2);

        //Total Cost(Buyer)
        ind_etcB = (((K + F)*D*EA)/((1-Ep)*(1-Ee1)*qind)) + ((cab*D*Ep*Ee2)/((1-Ep)*(1-Ee1))) +
(hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*qind)/(2*EA);

        //Total Cost(Vendor)
        ind_etcV = ((D/((1-Ep)*(1-Ee1)))*(((sv*EA)/qind) +cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci +
(hv/(2*P)*(1 +((Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA))*qind));
        ind_etcT = ind_etcV + ind_etcB;

    cost_diff = ind_etcT- opt_etc;
        printf("%-3.0f   %-14.2f   %-14.2f   %-14.2f   %-14.2f   %-2d   %-14.2f   %-14.2f   %-
14.2f\n",hv,qind,ind_etcB,ind_etcV,ind_etcT, opt_n,opt_q,opt_etc,cost_diff);
    }
    return 0;
}
```

4. The minimum Expected Total Cost for the integrated solution with respect to different buyer's inventory holding cost h_b.

Program Code: A2.4

```
#include <stdio.h>
#include <math.h>
int main()
{
    float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, bita=0.04, alfa=0.04, neno=0.04;
    float q1,qind,q3,q4,q5,qt,ind_etcV,ind_etcB,ind_etcT,cost_diff;
    float El, El2, Ee1, Ee2, Ee12, EA, EA2;
    float q,ETCq,ETCqv;
    float Ep, Ep2, Ee22;
    int n,opt_n=0;
    double A1,A2,A3,A4,A5,A6,A7,A8,A9;
    double B1,B2,B3,B4,B5,B6,etc;
    double c1,c2,c3,c4,c5,c6,cetc;
    double opt_q=0.0, opt_etc;

    printf("hb      Qb              ETCb(Qb)        ETCv(Qv)        Total Cost      n    Q(n)            ETC(n,Q(n))
Cost Reduction\n");
    for(hb=1;hb<=10;hb++)
    {
        Ep = bita/2;
        Ee1 = alfa/2;
        Ee2 = neno/2;

        //Square
```

```
        Ep2 = (bita*bita)/3;
        Ee12 = (alfa*alfa)/3;
        Ee22 = (neno*neno)/3;

        EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
        EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 +
2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;

        opt_n = 0;
        opt_q = 0.0;
        opt_etc = 99999999.0;

        for(n=1;n<=15;n++)
        {
            A1 = (sv + K + n*F);
            A2 = 2*n - n*n;
            A3 = (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA);
            A4 = (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2;
            A5 = ((A2 + A3 + A4) * hv)/(2*P);
            A6 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1)) / (2*EA2*D);
            A7 = A1 / ( A5 + A6);

            q = sqrt(A7);

            B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
            B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-
Ep)*(1-Ee1));
            B3 = (((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-
Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2)* hv)/ (2*P);
            B4 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1))/(2*EA2*D);
            B5 = ((B3 +B4)*q*D*EA)/(n*(1-Ep)*(1-Ee1));
            etc = B1 + B2 + B5;
            if (opt_etc > etc || n == 0 )
            {
                opt_n = n;
                opt_q = q;
                opt_etc = etc;
            }
        }

        //indipendant solution
        c1 = 2*(K + F)*D*EA2;
        c2 = hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1);
        qind = sqrt(c1/c2);

        //Total Cost(Buyer)
        ind_etcB = (((K + F)*D*EA)/((1-Ep)*(1-Ee1)*qind)) + ((cab*D*Ep*Ee2)/((1-Ep)*(1-Ee1))) +
(hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*qind)/(2*EA);

        //Total Cost(Vendor)
        ind_etcV = ((D/((1-Ep)*(1-Ee1)))*(((sv*EA)/qind) +cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci +
(hv/(2*P)*(1 +((Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA))*qind));
        ind_etcT = ind_etcV + ind_etcB;

    cost_diff = ind_etcT- opt_etc;
        printf("%-3.0f   %-14.2f   %-14.2f   %-14.2f   %-14.2f   %-2d   %-14.2f   %-14.2f   %-
14.2f\n",hb,qind,ind_etcB,ind_etcV,ind_etcT, opt_n,opt_q,opt_etc,cost_diff);
    }
    return 0;
}
```

5. The minimum Expected Total Cost for the integrated solution for different probabilities of defective items produced p which are uniformly distributed between 0 and β.

Program Code: A2.5

```
#include <stdio.h>
#include <math.h>
int main()
{
	float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, bita=0.04, alfa=0.04, neno=0.04;
	float q1,qind,q3,q4,q5,qt,ind_etcV,ind_etcB,ind_etcT,cost_diff;
	float El, El2, Ee1, Ee2, Ee12, EA, EA2;
	float q,ETCq,ETCqv;
	float Ep, Ep2, Ee22;
	int n,opt_n=0;
	double A1,A2,A3,A4,A5,A6,A7,A8,A9;
	double B1,B2,B3,B4,B5,B6,etc;
	double c1,c2,c3,c4,c5,c6,cetc;
	double opt_q=0.0, opt_etc;

	printf("bita   Qb          ETCb(Qb)       ETCv(Qv)       Total Cost      n    Q(n)          ETC(n,Q(n))
Cost Reduction\n");
	for(bita=0.00;bita<=1;bita=bita + 0.02)
	{
		Ep = bita/2;
		Ee1 = alfa/2;
		Ee2 = neno/2;

		//Square
		Ep2 = (bita*bita)/3;
		Ee12 = (alfa*alfa)/3;
		Ee22 = (neno*neno)/3;

		EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
		EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 +
2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;

		opt_n = 0;
		opt_q = 0.0;
		opt_etc = 99999999.0;

		for(n=1;n<=15;n++)
		{
			A1 = (sv + K + n*F);
			A2 = 2*n - n*n;
			A3 = (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA);
			A4 = (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2;
			A5 = ((A2 + A3 + A4) * hv)/(2*P);
			A6 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1)) / (2*EA2*D);
			A7 = A1 / ( A5 + A6);

			q = sqrt(A7);

			B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
			B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-
Ep)*(1-Ee1));
```

```
            B3 = (((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-
Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2)* hv)/ (2*P);
            B4 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1))/(2*EA2*D);
            B5 = ((B3 +B4)*q*D*EA)/(n*(1-Ep)*(1-Ee1));
            etc = B1 + B2 + B5;

            if (opt_etc > etc || n == 0 )
            {
                opt_n = n;
                opt_q = q;
                opt_etc = etc;
            }
        }

        //indipendant solution
        c1 = 2*(K + F)*D*EA2;
        c2 = hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1);
        qind = sqrt(c1/c2);

        //Total Cost(Buyer)
        ind_etcB = (((K + F)*D*EA)/((1-Ep)*(1-Ee1)*qind)) + ((cab*D*Ep*Ee2)/((1-Ep)*(1-Ee1))) +
(hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*qind)/(2*EA);

        //Total Cost(Vendor)
        ind_etcV = ((D/((1-Ep)*(1-Ee1)))*(((sv*EA)/qind) +cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci +
(hv/(2*P))*(1 +((Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA))*qind));
        ind_etcT = ind_etcV + ind_etcB;
    cost_diff = ind_etcT- opt_etc;
        printf("%-3.2f  %-14.2f  %-14.2f  %-14.2f  %-14.2f  %-2d  %-14.2f  %-14.2f  %-
14.2f\n",bita,qind,ind_etcB,ind_etcV,ind_etcT, opt_n,opt_q,opt_etc,cost_diff);
    }
    return 0;
}
```

6. The minimum Expected Total Cost for the integrated solution for different probabilities of type I inspection error e_1 which are uniformly distributed between 0 and λ.

Program Code: A2.6

```
#include <stdio.h>
#include <math.h>
int main()
{
    float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, bita=0.04, alfa=0.04, neno=0.04;
    float q1,qind,q3,q4,q5,qt,ind_etcV,ind_etcB,ind_etcT,cost_diff;
    float El, El2, Ee1, Ee2, Ee12, EA, EA2;
    float q,ETCq,ETCqv;
    float Ep, Ep2, Ee22;
    int n,opt_n=0;
    double A1,A2,A3,A4,A5,A6,A7,A8,A9;
    double B1,B2,B3,B4,B5,B6,etc;
    double c1,c2,c3,c4,c5,c6,cetc;
    double opt_q=0.0, opt_etc;

    printf("alfa   Qb          ETCb(Qb)       ETCv(Qv)       Total Cost      n    Q(n)          ETC(n,Q(n))
Cost Reduction\n");
```

```
	for(alfa=0.00;alfa<=1;alfa=alfa + 0.02)
	{
		Ep = bita/2;
		Ee1 = alfa/2;
		Ee2 = neno/2;

		//Square
		Ep2 = (bita*bita)/3;
		Ee12 = (alfa*alfa)/3;
		Ee22 = (neno*neno)/3;

		EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
		EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 +
2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;

		opt_n = 0;
		opt_q = 0.0;
		opt_etc = 99999999.0;

		for(n=1;n<=15;n++)
		{
			A1 = (sv + K + n*F);
			A2 = 2*n - n*n;
			A3 = (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA);
			A4 = (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2;
			A5 = ((A2 + A3 + A4) * hv)/(2*P);
			A6 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1)) / (2*EA2*D);
			A7 = A1 / ( A5 + A6);

			q = sqrt(A7);

			B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
			B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-
Ep)*(1-Ee1));
			B3 = (((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-
Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2)* hv)/ (2*P);
			B4 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1))/(2*EA2*D);
			B5 = ((B3 +B4)*q*D*EA)/(n*(1-Ep)*(1-Ee1));
			etc = B1 + B2 + B5;

			if (opt_etc > etc || n == 0 )
			{
				opt_n = n;
				opt_q = q;
				opt_etc = etc;
			}
		}

		//indipendant solution
		c1 = 2*(K + F)*D*EA2;
		c2 = hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1);
		qind = sqrt(c1/c2);

		//Total Cost(Buyer)
		ind_etcB = (((K + F)*D*EA)/((1-Ep)*(1-Ee1)*qind)) + ((cab*D*Ep*Ee2)/((1-Ep)*(1-Ee1))) +
(hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*qind)/(2*EA);

		//Total Cost(Vendor)
		ind_etcV = ((D/((1-Ep)*(1-Ee1)))*(((sv*EA)/qind) +cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci +
(hv/(2*P))*(1 +((Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA))*qind));
```

```
            ind_etcT = ind_etcV + ind_etcB;

        cost_diff = ind_etcT- opt_etc;
            printf("%-3.2f  %-14.2f  %-14.2f  %-14.2f  %-14.2f  %-2d  %-14.2f  %-14.2f  %-
14.2f\n",alfa,qind,ind_etcB,ind_etcV,ind_etcT, opt_n,opt_q,opt_etc,cost_diff);
        }
        return 0;
}
```

7. The minimum Expected Total Cost for the integrated solution for different probabilities of type II inspection error e_2 which are uniformly distributed between 0 and η.

Program Code: A2.7

```
#include <stdio.h>
#include <math.h>
int main()
{
        float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, bita=0.04, alfa=0.04, neno=0.04;
        float q1,qind,q3,q4,q5,qt,ind_etcV,ind_etcB,ind_etcT,cost_diff;
        float El, El2, Ee1, Ee2, Ee12, EA, EA2;
        float q,ETCq,ETCqv;
        float Ep, Ep2, Ee22;
        int n,opt_n=0;
        double A1,A2,A3,A4,A5,A6,A7,A8,A9;
        double B1,B2,B3,B4,B5,B6,etc;
        double c1,c2,c3,c4,c5,c6,cetc;
        double opt_q=0.0, opt_etc;

        printf("neno   Qb             ETCb(Qb)       ETCv(Qv)       Total Cost      n    Q(n)           ETC(n,Q(n))
Cost Reduction\n");
        for(neno=0.00;neno<=1;neno=neno + 0.02)
        {
            Ep = bita/2;
            Ee1 = alfa/2;
            Ee2 = neno/2;

            //Square
            Ep2 = (bita*bita)/3;
            Ee12 = (alfa*alfa)/3;
            Ee22 = (neno*neno)/3;

            EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
            EA2 = 1 - 2*Ep  +  2*Ep*Ee2  -  2*Ee1  +  2*Ep*Ee1  +  Ep2  -  2*Ep2*Ee2  +  Ep2*Ee22  +
2*Ep*Ee1  -  2*Ep*Ee1*Ee2  -  2*Ep2*Ee1  +  2*Ep2*Ee1*Ee2  +  Ee12  -  2*Ep*Ee12  +  Ep2*Ee12;

            opt_n = 0;
            opt_q = 0.0;
            opt_etc = 99999999.0;

            for(n=1;n<=15;n++)
            {
                A1 = (sv + K + n*F);
                A2 = 2*n - n*n;
                A3 = (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA);
                A4 = (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2;
```

```
            A5 = ((A2 + A3 + A4) * hv)/(2*P);
            A6 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1)) / (2*EA2*D);
            A7 = A1 / ( A5 + A6);

            q = sqrt(A7);

            B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
            B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-
Ep)*(1-Ee1));
            B3 = (((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-
Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2)* hv)/ (2*P);
            B4 = (n*hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1))/(2*EA2*D);
            B5 = ((B3 +B4)*q*D*EA)/(n*(1-Ep)*(1-Ee1));
            etc = B1 + B2 + B5;

            if (opt_etc > etc || n == 0 )
            {
                opt_n = n;
                opt_q = q;
                opt_etc = etc;
            }
        }

        //indipendant solution
        c1 = 2*(K + F)*D*EA2;
        c2 = hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*(1-Ep)*(1-Ee1);
        qind = sqrt(c1/c2);

        //Total Cost(Buyer)
        ind_etcB = (((K + F)*D*EA)/((1-Ep)*(1-Ee1)*qind)) + ((cab*D*Ep*Ee2)/((1-Ep)*(1-Ee1))) +
(hb*(1-(Ep*(1-2*Ee2)+(1-Ep)*Ee1))*qind)/(2*EA);

        //Total Cost(Vendor)
        ind_etcV = ((D/((1-Ep)*(1-Ee1)))*(((sv*EA)/qind) +cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci +
(hv/(2*P))*(1 +((Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA))*qind));
        ind_etcT = ind_etcV + ind_etcB;

    cost_diff = ind_etcT- opt_etc;
        printf("%-3.2f   %-14.2f   %-14.2f   %-14.2f   %-14.2f   %-2d   %-14.2f   %-14.2f   %-
14.2f\n",neno,qind,ind_etcB,ind_etcV,ind_etcT, opt_n,opt_q,opt_etc,cost_diff);
    }
    return 0;
}
```

Appendix A3

This section presents code of C programs used in mathematical model "An integrated single-vendor, single-buyer inventory model for imperfect quality production, imperfect inspection at vendor site and with backorder"

1. Calculation of the minimum Expected Total Cost for the integrated solution with respect to lot size Q and number of lots n.

Program Code: A3.1

```
#include <stdio.h>
#include <math.h>
int main()
{
	float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100, cab=200, cav=300, b=10, bita=0.04 ;
	float q1,q2,q3,q4,q5,qt;
	float El, El2, Ee1, Ee2, Ee12, EA, EA2;
	float q,ETCq,ETCqv;
	float Ep, Ep2, Ee22;
	int n;
	double A1,A2,A3,A4,A5,A6,A7,A8,A9;
	double B1,B2,B3,B4,B5,B6,bkorder,etc;

	Ep = bita/2;
	Ee1 = bita/2;
	Ee2 = bita/2;

	//Square
	Ep2 = (bita*bita)/3;
	Ee12 = (bita*bita)/3;
	Ee22 = (bita*bita)/3;

	EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
	EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 + 2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;

	//printf("EA = %-f\t\tEA2=%-f\n",EA,EA2);

	printf("n      Q(n)        Backorder    ETC(n,Q(n))\n");
	for(n=1;n<=15;n++)
	{
		A1 = (sv + K + n*F)*D;
		A2 = ((n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA));
		A3 = ((n*n*(Ep*(1-Ee2) + (1-Ep)*Ee1)* (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA2);
		A4 = ((hv*D)/(2*P))*((2*n - n*n) + A2 + A3);
		A5 = (hb/2) * (n + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
		A6 = (n*hb*hb)/(2*(b+hb));
		A7 = A4 + A5 - A6;

		q = sqrt(A1/A7);

		B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
```

```
		B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-Ep)*(1-Ee1));
		B3 = (hb*hb*EA*q)/(2*(b+hb)*(1-Ep)*(1-Ee1));
		B4 = ((D*q*EA)/(2*n*(1-Ep)*(1-Ee1)))*((hv/P)*((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2) + (hb/D)*(n +((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2) );
		etc = B1 + B2 - B3 + B4;

		bkorder = q * (hb/(b+hb));
		printf("%-4d   %-10.4f   %-10.4f   %-10.4f\n",n,q,bkorder,etc);
	}
	return 0;
}
```

2. The minimum Expected Total Cost for the integrated solution with respect to different freight cost F.

Program Code: A3.2

```
#include <stdio.h>
#include <math.h>
int main()
{
	float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100, cab=200, cav=300, b=10, bita=0.04 ;
	float q1,q2,q3,q4,q5,qt;
	float El, El2, Ee1, Ee2, Ee12, EA, EA2;
	float q,ETCq,ETCqv;
	float Ep, Ep2, Ee22;
	int n, opt_n;
	double A1,A2,A3,A4,A5,A6,A7,A8,A9;
	double B1,B2,B3,B4,bkorder,etc;
	double x1,x2,ind_q,y1,y2,y3,y4,ind_etcB,ind_etcV,ind_bkorder,cost_diff;
	double opt_q=0.0, opt_etc=0.0,opt_bkorder=0.0;

	Ep = bita/2;
	Ee1 = bita/2;
	Ee2 = bita/2;

	//Square
	Ep2 = (bita*bita)/3;
	Ee12 = (bita*bita)/3;
	Ee22 = (bita*bita)/3;

	EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
	EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 + 2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;
	printf("F	Qb	ETCb(Qb)	ETCv(Qv)	ETC(Q*)	Backorder	n	Q(n)	ETC(n,Q(n))  Cost Reduction\n");
	for(F=5;F<=100;F=F+5)
	{
		opt_n = 0;
		opt_q = 0.0;
		opt_etc = 9999999.0;
		opt_bkorder = 0.0;
		x1 = 2*(K+F)*D;
		x2 = hb*(1+(b/(b+hb))+(((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
		ind_q = sqrt(x1/x2);
```

```
		ind_bkorder = (hb*ind_q)/(b+hb);

		y1 = ((K+F)*D*EA) / ((1-Ep)*(1-Ee1)*ind_q);
		y2 = (b*hb*hb*EA*ind_q) / (2*(b+hb)*(b+hb)*(1-Ep)*(1-Ee1));
		y3 = (cab*D*Ep*Ee2) / ((1-Ep)*(1-Ee1));
		y4 = (((b*b)/((b+hb)*(b+hb))) + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2)) * ((hb*EA*ind_q)/((1-Ep)*(1-Ee1)));
		ind_etcB = y1 + y2 + y3 + y4;

		y1 = (sv*D*EA)/((1-Ep)*(1-Ee1)*ind_q);
		y2 = (D/((1-Ep)*(1-Ee1))) * (cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci);
		y3 = ((hv*D*ind_q) / (2*P*(1-Ep)*(1-Ee1))) * (EA + ((Ep*(1-Ee2) + (1-Ep)*Ee1) * (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA);
		ind_etcV = y1 + y2 + y3;
		//printf("F=%3.0f  y1=%f  y2=%f  y3=%f\n",F,y1,y2,y3);

		for(n=1;n<=15;n++)
		{
			A1 = (sv + K + n*F)*D;
			A2 = ((n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA));
			A3 = ((n*n*(Ep*(1-Ee2) + (1-Ep)*Ee1)* (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA2);
			A4 = ((hv*D)/(2*P))*((2*n - n*n) + A2 + A3);
			A5 = (hb/2) * (n + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
			A6 = (n*hb*hb)/(2*(b+hb));
			A7 = A4 + A5 - A6;

			q = sqrt(A1/A7);

			B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
			B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-Ep)*(1-Ee1));
			B3 = (hb*hb*EA*q)/(2*(b+hb)*(1-Ep)*(1-Ee1));
			B4 = ((D*q*EA)/(2*n*(1-Ep)*(1-Ee1)))*((hv/P)*((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2) + (hb/D)*(n +((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2) );
			etc = B1 + B2 - B3 + B4;

			bkorder = q * (hb/(b+hb));
			if (etc < opt_etc)
			{
				opt_n = n;
				opt_q = q;
				opt_etc = etc;
				opt_bkorder = bkorder;
			}
		}
		cost_diff = ind_etcB + ind_etcV - opt_etc;
		//printf("F = %-3f   ind_Q* = %-10.4f   ind_B3 = %-10.4f   ind_ETC(B)* = %-10.4f   ind_ETC(V)* = %-10.4f   ind_ETC(T)* = %-10.4f   n=%d   Q* = %-10.4f   B3 = %-10.4f   ETC = %-10.4f   Cost_red = %-10.4f\n",F,ind_q,ind_bkorder,ind_etcB,ind_etcV,ind_etcB + ind_etcV,opt_n,opt_q,opt_bkorder,opt_etc,cost_diff);
		printf("%-3.0f  %-10.2f  %-10.2f  %-10.2f  %-10.2f  %-10.2f  %-2d  %-10.2f  %-10.2f  %-10.2f\n",F,ind_q,ind_etcB,ind_etcV, ind_etcB+ind_etcV,ind_bkorder, opt_n,opt_q,opt_etc,cost_diff);
	}
	return 0;
}
```

3. The minimum Expected Total Cost for the integrated solution with respect to different vendor's inventory holding cost h_v.

Program Code: A3.3

```
#include <stdio.h>
#include <math.h>
int main()
{
	float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, bita=0.04 ;
	float q1,q2,q3,q4,q5,qt;
	float El, El2, Ee1, Ee2, Ee12, EA, EA2;
	float q,ETCq,ETCqv;
	float Ep, Ep2, Ee22;
	int n, opt_n;
	double A1,A2,A3,A4,A5,A6,A7,A8,A9;
	double B1,B2,B3,B4,bkorder,etc;
	double x1,x2,ind_q,y1,y2,y3,y4,ind_etcB,ind_etcV,ind_bkorder,cost_diff;
	double opt_q=0.0, opt_etc=0.0,opt_bkorder=0.0;

	Ep = bita/2;
	Ee1 = bita/2;
	Ee2 = bita/2;

	//Square
	Ep2 = (bita*bita)/3;
	Ee12 = (bita*bita)/3;
	Ee22 = (bita*bita)/3;

	EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
	EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 + 2*Ep*Ee1
- 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;
	printf("hv   Qb        ETCb(Qb)   ETCv(Qv)   ETC(Q*)    backorder   n   Q(n)       ETC(n,Q(n))
Cost Reduction\n");
	for(hv=1;hv<=10;hv++)
	{
		opt_n = 0;
		opt_q = 0.0;
		opt_etc = 9999999.0;
		opt_bkorder = 0.0;
		x1 = 2*(K+F)*D;
		x2 = hb*(1+(b/(b+hb))+(((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
		ind_q = sqrt(x1/x2);
		ind_bkorder = (hb*ind_q)/(b+hb);

		y1 = ((K+F)*D*EA) / ((1-Ep)*(1-Ee1)*ind_q);
		y2 = (b*hb*hb*EA*ind_q) / (2*(b+hb)*(b+hb)*(1-Ep)*(1-Ee1));
		y3 = (cab*D*Ep*Ee2) / ((1-Ep)*(1-Ee1));
		y4 = (((b*b)/((b+hb)*(b+hb))) + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2)) * ((hb*EA*ind_q)/((1-Ep)*(1-
Ee1)));
		ind_etcB = y1 + y2 + y3 + y4;

		y1 = (sv*D*EA)/((1-Ep)*(1-Ee1)*ind_q);
		y2 = (D/((1-Ep)*(1-Ee1))) * (cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci);
		y3 = ((hv*D*ind_q) / (2*P*(1-Ep)*(1-Ee1))) * (EA + ((Ep*(1-Ee2) + (1-Ep)*Ee1) * (Ep*(1-Ee2) +
(1-Ep)*Ee1)) / EA);
		ind_etcV = y1 + y2 + y3;
		//printf("F=%3.0f  y1=%f  y2=%f  y3=%f\n",F,y1,y2,y3);
```

```
	for(n=1;n<=15;n++)
	{
		A1 = (sv + K + n*F)*D;
		A2 = ((n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA));
		A3 = ((n*n*(Ep*(1-Ee2) + (1-Ep)*Ee1)* (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA2);
		A4 = ((hv*D)/(2*P))*((2*n - n*n) + A2 + A3);
		A5 = (hb/2) * (n + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
		A6 = (n*hb*hb)/(2*(b+hb));
		A7 = A4 + A5 - A6;

		q = sqrt(A1/A7);

		B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
		B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-
Ep)*(1-Ee1));
		B3 = (hb*hb*EA*q)/(2*(b+hb)*(1-Ep)*(1-Ee1));
		B4 = ((D*q*EA)/(2*n*(1-Ep)*(1-Ee1)))*((hv/P)*((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-
Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2) + (hb/D)*(n +((1-Ep)*(1-
Ee1)*Ep*Ee2)/EA2) );
		etc = B1 + B2 - B3 + B4;

		bkorder = q * (hb/(b+hb));
		if (etc < opt_etc)
		{
			opt_n = n;
			opt_q = q;
			opt_etc = etc;
			opt_bkorder = bkorder;
		}
	}
	cost_diff = ind_etcB + ind_etcV - opt_etc;
	//printf("F = %-3f   ind_Q* = %-10.4f   ind_B3 = %-10.4f   ind_ETC(B)* = %-10.4f   ind_ETC(V)* =
%-10.4f   ind_ETC(T)* = %-10.4f   n=%d   Q* = %-10.4f   B3 = %-10.4f   ETC = %-10.4f   Cost_red = %-
10.4f\n",F,ind_q,ind_bkorder,ind_etcB,ind_etcV,ind_etcB +
ind_etcV,opt_n,opt_q,opt_bkorder,opt_etc,cost_diff);
	printf("%-3.0f   %-10.2f   %-10.2f   %-10.2f   %-10.2f   %-10.2f   %-2d   %-10.2f   %-10.2f   %-
10.2f\n",hv,ind_q,ind_etcB,ind_etcV,ind_etcB+ind_etcV,ind_bkorder, opt_n,opt_q,opt_etc,cost_diff);
    }
    return 0;
}
```

4. The minimum Expected Total Cost for the integrated solution with respect to different buyer's inventory holding cost h_b.

Program Code: A3.4

```
#include <stdio.h>
#include <math.h>
int main()
{
	float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, bita=0.04 ;
	float q1,q2,q3,q4,q5,qt;
	float El, El2, Ee1, Ee2, Ee12, EA, EA2;
	float q,ETCq,ETCqv;
	float Ep, Ep2, Ee22;
	int n, opt_n;
```

```
	double A1,A2,A3,A4,A5,A6,A7,A8,A9;
	double B1,B2,B3,B4,bkorder,etc;
	double x1,x2,ind_q,y1,y2,y3,y4,ind_etcB,ind_etcV,ind_bkorder,cost_diff;
	double opt_q=0.0, opt_etc=0.0,opt_bkorder=0.0;

	Ep = bita/2;
	Ee1 = bita/2;
	Ee2 = bita/2;

	//Square
	Ep2 = (bita*bita)/3;
	Ee12 = (bita*bita)/3;
	Ee22 = (bita*bita)/3;

	EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
	EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 +
2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;
	printf("hb	Qb	ETCb(Qb)	ETCv(Qv)	ETC(Q*)	backorder	n	Q(n)
ETC(n,Q(n))  Cost Reduction\n");
	for(hb=1;hb<=10;hb++)
	{
		opt_n = 0;
		opt_q = 0.0;
		opt_etc = 9999999.0;
		opt_bkorder = 0.0;
		x1 = 2*(K+F)*D;
		x2 = hb*(1+(b/(b+hb))+(((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
		ind_q = sqrt(x1/x2);
		ind_bkorder = (hb*ind_q)/(b+hb);

		y1 = ((K+F)*D*EA) / ((1-Ep)*(1-Ee1)*ind_q);
		y2 = (b*hb*hb*EA*ind_q) / (2*(b+hb)*(b+hb)*(1-Ep)*(1-Ee1));
		y3 = (cab*D*Ep*Ee2) / ((1-Ep)*(1-Ee1));
		y4 = (((b*b)/((b+hb)*(b+hb))) + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2)) * ((hb*EA*ind_q)/((1-
Ep)*(1-Ee1)));
		ind_etcB = y1 + y2 + y3 + y4;

		y1 = (sv*D*EA)/((1-Ep)*(1-Ee1)*ind_q);
		y2 = (D/((1-Ep)*(1-Ee1))) * (cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci);
		y3 = ((hv*D*ind_q) / (2*P*(1-Ep)*(1-Ee1))) * (EA + ((Ep*(1-Ee2) + (1-Ep)*Ee1) * (Ep*(1-
Ee2) + (1-Ep)*Ee1)) / EA);
		ind_etcV = y1 + y2 + y3;
		//printf("F=%3.0f  y1=%f  y2=%f  y3=%f\n",F,y1,y2,y3);

		for(n=1;n<=15;n++)
		{
			A1 = (sv + K + n*F)*D;
			A2 = ((n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA));
			A3 = ((n*n*(Ep*(1-Ee2) + (1-Ep)*Ee1)* (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA2);
			A4 = ((hv*D)/(2*P))*((2*n - n*n) + A2 + A3);
			A5 = (hb/2) * (n + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
			A6 = (n*hb*hb)/(2*(b+hb));
			A7 = A4 + A5 - A6;

			q = sqrt(A1/A7);

			B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
			B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D)
/(n*(1-Ep)*(1-Ee1));
			B3 = (hb*hb*EA*q)/(2*(b+hb)*(1-Ep)*(1-Ee1));
```

```
                    B4 = ((D*q*EA)/(2*n*(1-Ep)*(1-Ee1)))*((hv/P)*((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2) + (hb/D)*(n +((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2) );
                    etc = B1 + B2 - B3 + B4;

                    bkorder = q * (hb/(b+hb));
                    if (etc < opt_etc)
                    {
                            opt_n = n;
                            opt_q = q;
                            opt_etc = etc;
                            opt_bkorder = bkorder;
                    }

                }
                cost_diff = ind_etcB + ind_etcV - opt_etc;
                //printf("F = %-3f   ind_Q* = %-10.4f   ind_B3 = %-10.4f   ind_ETC(B)* = %-10.4f ind_ETC(V)* = %-10.4f   ind_ETC(T)* = %-10.4f   n=%d   Q* = %-10.4f   B3 = %-10.4f   ETC = %-10.4f Cost_red = %-10.4f\n",F,ind_q,ind_bkorder,ind_etcB,ind_etcV,ind_etcB + ind_etcV,opt_n,opt_q,opt_bkorder,opt_etc,cost_diff);
                printf("%-3.0f %-10.2f %-10.2f %-10.2f %-10.2f %-10.2f %-2d %-10.2f %-10.2f %-10.2f\n",hb,ind_q,ind_etcB,ind_etcV,ind_etcB+ind_etcV,ind_bkorder, opt_n,opt_q,opt_etc,cost_diff);
        }
        return 0;
}
```

5. The minimum Expected Total Cost for the integrated solution for different probabilities of defective items produced p which are uniformly distributed between 0 and β.

Program Code: A3.5

```
#include <stdio.h>
#include <math.h>
int main()
{
        float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100, cab=200, cav=300, b=10, bita=0.04, alfa=0.04, neno=0.04;
        float q1,q2,q3,q4,q5,qt;
        float El, El2, Ee1, Ee2, Ee12, EA, EA2;
        float q,ETCq,ETCqv;
        float Ep, Ep2, Ee22;
        int n, opt_n;
        double A1,A2,A3,A4,A5,A6,A7,A8,A9;
        double B1,B2,B3,B4,bkorder,etc;
        double x1,x2,ind_q,y1,y2,y3,y4,ind_etcB,ind_etcV,ind_bkorder,cost_diff;
        double opt_q=0.0, opt_etc=0.0,opt_bkorder=0.0;

        printf("bita   Qb          ETCb(Qb)      ETCv(Qv)      ETC(Q*)       backorder   n   Q(n) ETC(n,Q(n))  Cost Reduction\n");
        for(bita=0.00;bita<=1;bita=bita + 0.02)
        {
              Ep = bita/2;
              Ee1 = alfa/2;
              Ee2 = neno/2;
```

```
//Square
Ep2 = (bita*bita)/3;
Ee12 = (alfa*alfa)/3;
Ee22 = (neno*neno)/3;

EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 + 2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;

opt_n = 0;
opt_q = 0.0;
opt_etc = 9999999.0;
opt_bkorder = 0.0;
x1 = 2*(K+F)*D;
x2 = hb*(1+(b/(b+hb))+(((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
ind_q = sqrt(x1/x2);
ind_bkorder = (hb*ind_q)/(b+hb);

y1 = ((K+F)*D*EA) / ((1-Ep)*(1-Ee1)*ind_q);
y2 = (b*hb*hb*EA*ind_q) / (2*(b+hb)*(b+hb)*(1-Ep)*(1-Ee1));
y3 = (cab*D*Ep*Ee2) / ((1-Ep)*(1-Ee1));
y4 = (((b*b)/((b+hb)*(b+hb))) + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2)) * ((hb*EA*ind_q)/((1-Ep)*(1-Ee1)));
ind_etcB = y1 + y2 + y3 + y4;

y1 = (sv*D*EA)/((1-Ep)*(1-Ee1)*ind_q);
y2 = (D/((1-Ep)*(1-Ee1))) * (cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci);
y3 = ((hv*D*ind_q) / (2*P*(1-Ep)*(1-Ee1))) * (EA + ((Ep*(1-Ee2) + (1-Ep)*Ee1) * (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA);
ind_etcV = y1 + y2 + y3;
//printf("F=%3.0f  y1=%f  y2=%f  y3=%f\n",F,y1,y2,y3);

for(n=1;n<=15;n++)
{
    A1 = (sv + K + n*F)*D;
    A2 = ((n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA));
    A3 = ((n*n*(Ep*(1-Ee2) + (1-Ep)*Ee1)* (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA2);
    A4 = ((hv*D)/(2*P))*((2*n - n*n) + A2 + A3);
    A5 = (hb/2) * (n + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
    A6 = (n*hb*hb)/(2*(b+hb));
    A7 = A4 + A5 - A6;

    q = sqrt(A1/A7);

    B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
    B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-Ep)*(1-Ee1));
    B3 = (hb*hb*EA*q)/(2*(b+hb)*(1-Ep)*(1-Ee1));
    B4 = ((D*q*EA)/(2*n*(1-Ep)*(1-Ee1)))*((hv/P)*((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2) + (hb/D)*(n +((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2) );
    etc = B1 + B2 - B3 + B4;

    bkorder = q * (hb/(b+hb));
    if (etc < opt_etc)
    {
        opt_n = n;
        opt_q = q;
        opt_etc = etc;
```

```
                        opt_bkorder = bkorder;
                }

            }
            cost_diff = ind_etcB + ind_etcV - opt_etc;
            //printf("F = %-3f    ind_Q* = %-10.4f    ind_B3 = %-10.4f    ind_ETC(B)* = %-10.4f
ind_ETC(V)* = %-10.4f   ind_ETC(T)* = %-10.4f   n=%d   Q* = %-10.4f   B3 = %-10.4f   ETC = %-10.4f
Cost_red    =    %-10.4f\n",F,ind_q,ind_bkorder,ind_etcB,ind_etcV,ind_etcB    +
ind_etcV,opt_n,opt_q,opt_bkorder,opt_etc,cost_diff);
            printf("%-3.2f  %-10.2f  %-10.2f  %-10.2f  %-10.2f  %-10.2f  %-2d  %-10.2f  %-10.2f  %-
10.2f\n",bita,ind_q,ind_etcB,ind_etcV,ind_etcB+ind_etcV,ind_bkorder, opt_n,opt_q,opt_etc,cost_diff);
        }
        return 0;
}
```

6. The minimum Expected Total Cost for the integrated solution for different probabilities of type I inspection error e_1 which are uniformly distributed between 0 and λ.

Program Code: A3.6

```
#include <stdio.h>
#include <math.h>
int main()
{
        float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, lamda=0.04, alfa=0.04, neno=0.04;
        float q1,q2,q3,q4,q5,qt;
        float El, El2, Ee1, Ee2, Ee12, EA, EA2;
        float q,ETCq,ETCqv;
        float Ep, Ep2, Ee22;
        int n, opt_n;
        double A1,A2,A3,A4,A5,A6,A7,A8,A9;
        double B1,B2,B3,B4,bkorder,etc;
        double x1,x2,ind_q,y1,y2,y3,y4,ind_etcB,ind_etcV,ind_bkorder,cost_diff;
        double opt_q=0.0, opt_etc=0.0,opt_bkorder=0.0;

        printf("Lamda   Qb          ETCb(Qb)     ETCv(Qv)     ETC(Q*)      backorder    n     Q(n)
ETC(n,Q(n))  Cost Reduction\n");
        for(alfa=0.00;alfa<=1;alfa=alfa + 0.02)
        {
            Ep = lamda/2;
            Ee1 = alfa/2;
            Ee2 = neno/2;
            //Square
            Ep2 = (lamda*lamda)/3;
            Ee12 = (alfa*alfa)/3;
            Ee22 = (neno*neno)/3;
            EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
            EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 +
2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;
            opt_n = 0;
            opt_q = 0.0;
            opt_etc = 9999999.0;
            opt_bkorder = 0.0;
```

```
		x1 = 2*(K+F)*D;
		x2 = hb*(1+(b/(b+hb))+(((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
		ind_q = sqrt(x1/x2);
		ind_bkorder = (hb*ind_q)/(b+hb);

		y1 = ((K+F)*D*EA) / ((1-Ep)*(1-Ee1)*ind_q);
		y2 = (b*hb*hb*EA*ind_q) / (2*(b+hb)*(b+hb)*(1-Ep)*(1-Ee1));
		y3 = (cab*D*Ep*Ee2) / ((1-Ep)*(1-Ee1));
		y4 = (((b*b)/((b+hb)*(b+hb))) + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2)) * ((hb*EA*ind_q)/((1-Ep)*(1-Ee1)));
		ind_etcB = y1 + y2 + y3 + y4;

		y1 = (sv*D*EA)/((1-Ep)*(1-Ee1)*ind_q);
		y2 = (D/((1-Ep)*(1-Ee1))) * (cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci);
		y3 = ((hv*D*ind_q) / (2*P*(1-Ep)*(1-Ee1))) * (EA + ((Ep*(1-Ee2) + (1-Ep)*Ee1) * (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA);
		ind_etcV = y1 + y2 + y3;
		//printf("F=%3.0f  y1=%f  y2=%f  y3=%f\n",F,y1,y2,y3);

		for(n=1;n<=15;n++)
		{
			A1 = (sv + K + n*F)*D;
			A2 = ((n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA));
			A3 = ((n*n*(Ep*(1-Ee2) + (1-Ep)*Ee1)* (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA2);
			A4 = ((hv*D)/(2*P))*((2*n - n*n) + A2 + A3);
			A5 = (hb/2) * (n + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
			A6 = (n*hb*hb)/(2*(b+hb));
			A7 = A4 + A5 - A6;

			q = sqrt(A1/A7);

			B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
			B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-Ep)*(1-Ee1));
			B3 = (hb*hb*EA*q)/(2*(b+hb)*(1-Ep)*(1-Ee1));
			B4 = ((D*q*EA)/(2*n*(1-Ep)*(1-Ee1)))*((hv/P)*((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2) + (hb/D)*(n +((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2) );
			etc = B1 + B2 - B3 + B4;

			bkorder = q * (hb/(b+hb));
			if (etc < opt_etc)
			{
				opt_n = n;
				opt_q = q;
				opt_etc = etc;
				opt_bkorder = bkorder;
			}
		}
		cost_diff = ind_etcB + ind_etcV - opt_etc;
		//printf("F = %-3f   ind_Q* = %-10.4f   ind_B3 = %-10.4f   ind_ETC(B)* = %-10.4f   ind_ETC(V)* = %-10.4f   ind_ETC(T)* = %-10.4f   n=%d   Q* = %-10.4f   B3 = %-10.4f   ETC = %-10.4f   Cost_red = %-10.4f\n",F,ind_q,ind_bkorder,ind_etcB,ind_etcV,ind_etcB + ind_etcV,opt_n,opt_q,opt_bkorder,opt_etc,cost_diff);
		printf("%-3.2f  %-10.2f  %-10.2f  %-10.2f  %-10.2f  %-10.2f  %-2d  %-10.2f  %-10.2f  %-10.2f\n",alfa,ind_q,ind_etcB,ind_etcV,ind_etcB+ind_etcV,ind_bkorder, opt_n,opt_q,opt_etc,cost_diff);
	}
	return 0;
}
```

7. The minimum Expected Total Cost for the integrated solution for different probabilities of type II inspection error e_2 which are uniformly distributed between 0 and η.

Program Code: A3.7

```
#include <stdio.h>
#include <math.h>
int main()
{
        float P=160000, D=50000, x=175200, sv =300, K=100, hv=2, hb=5, F=25, ci=0.5, cw=50, cr=100,
cab=200, cav=300, b=10, bita=0.04, alfa=0.04, neno=0.04;
        float q1,q2,q3,q4,q5,qt;
        float El, El2, Ee1, Ee2, Ee12, EA, EA2;
        float q,ETCq,ETCqv;
        float Ep, Ep2, Ee22;
        int n, opt_n;
        double A1,A2,A3,A4,A5,A6,A7,A8,A9;
        double B1,B2,B3,B4,bkorder,etc;
        double x1,x2,ind_q,y1,y2,y3,y4,ind_etcB,ind_etcV,ind_bkorder,cost_diff;
        double opt_q=0.0, opt_etc=0.0,opt_bkorder=0.0;

        printf("neno    Qb          ETCb(Qb)     ETCv(Qv)     ETC(Q*)     Backorder     n     Q(n)
ETC(n,Q(n))  Cost Reduction\n");
        for(neno=0.00;neno<=1;neno=neno + 0.02)
        {
            Ep = bita/2;
            Ee1 = alfa/2;
            Ee2 = neno/2;

            //Square
            Ep2 = (bita*bita)/3;
            Ee12 = (alfa*alfa)/3;
            Ee22 = (neno*neno)/3;

            EA = 1 - (Ep*(1-Ee2) + (1-Ep)*Ee1);
            EA2 = 1 - 2*Ep + 2*Ep*Ee2 - 2*Ee1 + 2*Ep*Ee1 + Ep2 - 2*Ep2*Ee2 + Ep2*Ee22 +
2*Ep*Ee1 - 2*Ep*Ee1*Ee2 - 2*Ep2*Ee1 + 2*Ep2*Ee1*Ee2 + Ee12 - 2*Ep*Ee12 + Ep2*Ee12;

            opt_n = 0;
            opt_q = 0.0;
            opt_etc = 9999999.0;
            opt_bkorder = 0.0;
            x1 = 2*(K+F)*D;
            x2 = hb*(1+(b/(b+hb))+(((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
            ind_q = sqrt(x1/x2);
            ind_bkorder = (hb*ind_q)/(b+hb);

            y1 = ((K+F)*D*EA) / ((1-Ep)*(1-Ee1)*ind_q);
            y2 = (b*hb*hb*EA*ind_q) / (2*(b+hb)*(b+hb)*(1-Ep)*(1-Ee1));
            y3 = (cab*D*Ep*Ee2) / ((1-Ep)*(1-Ee1));
            y4 = (((b*b)/((b+hb)*(b+hb))) + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2)) * ((hb*EA*ind_q)/((1-
Ep)*(1-Ee1)));
            ind_etcB = y1 + y2 + y3 + y4;

            y1 = (sv*D*EA)/((1-Ep)*(1-Ee1)*ind_q);
            y2 = (D/((1-Ep)*(1-Ee1))) * (cw*Ep + cr*(1-Ep)*Ee1 + cav*Ep*Ee2 + ci);
```

```
		y3 = ((hv*D*ind_q) / (2*P*(1-Ep)*(1-Ee1))) * (EA + ((Ep*(1-Ee2) + (1-Ep)*Ee1) * (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA);
		ind_etcV = y1 + y2 + y3;
		//printf("F=%3.0f  y1=%f  y2=%f  y3=%f\n",F,y1,y2,y3);

		for(n=1;n<=15;n++)
		{
			A1 = (sv + K + n*F)*D;
			A2 = ((n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA));
			A3 = ((n*n*(Ep*(1-Ee2) + (1-Ep)*Ee1)* (Ep*(1-Ee2) + (1-Ep)*Ee1)) / EA2);
			A4 = ((hv*D)/(2*P))*((2*n - n*n) + A2 + A3);
			A5 = (hb/2) * (n + (((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2));
			A6 = (n*hb*hb)/(2*(b+hb));
			A7 = A4 + A5 - A6;

			q = sqrt(A1/A7);

			B1 = ((sv + K + n*F)*(D*EA))/(n*q*(1-Ep)*(1-Ee1));
			B2 = (((n*cw*Ep) + (n*cr*(1-Ep)*Ee1) + (n*cav*Ep*Ee2) + n*ci + n*cab*Ep*Ee2)*D) /(n*(1-Ep)*(1-Ee1));
			B3 = (hb*hb*EA*q)/(2*(b+hb)*(1-Ep)*(1-Ee1));
			B4 = ((D*q*EA)/(2*n*(1-Ep)*(1-Ee1)))*((hv/P)*((2*n - n*n) + (n*(n-1)*P*(1-Ep)*(1-Ee1))/(D*EA) + (n*n*(Ep*(1-Ee2)+(1-Ep)*Ee1)*(Ep*(1-Ee2)+(1-Ep)*Ee1))/EA2) + (hb/D)*(n +((1-Ep)*(1-Ee1)*Ep*Ee2)/EA2) );
			etc = B1 + B2 - B3 + B4;

			bkorder = q * (hb/(b+hb));
			if (etc < opt_etc)
			{
				opt_n = n;
				opt_q = q;
				opt_etc = etc;
				opt_bkorder = bkorder;
			}
		}
		cost_diff = ind_etcB + ind_etcV - opt_etc;
		//printf("F = %-3f   ind_Q* = %-10.4f   ind_B3 = %-10.4f   ind_ETC(B)* = %-10.4f  ind_ETC(V)* = %-10.4f   ind_ETC(T)* = %-10.4f   n=%d   Q* = %-10.4f   B3 = %-10.4f   ETC = %-10.4f  Cost_red = %-10.4f\n",F,ind_q,ind_bkorder,ind_etcB,ind_etcV,ind_etcB + ind_etcV,opt_n,opt_q,opt_bkorder,opt_etc,cost_diff);
		printf("%-3.2f  %-10.2f  %-10.2f  %-10.2f  %-10.2f  %-10.2f  %-2d  %-10.2f  %-10.2f  %-10.2f\n",neno,ind_q,ind_etcB,ind_etcV, ind_etcB+ind_etcV, ind_bkorder, opt_n,opt_q,opt_etc,cost_diff);
	}
	return 0;
}
```